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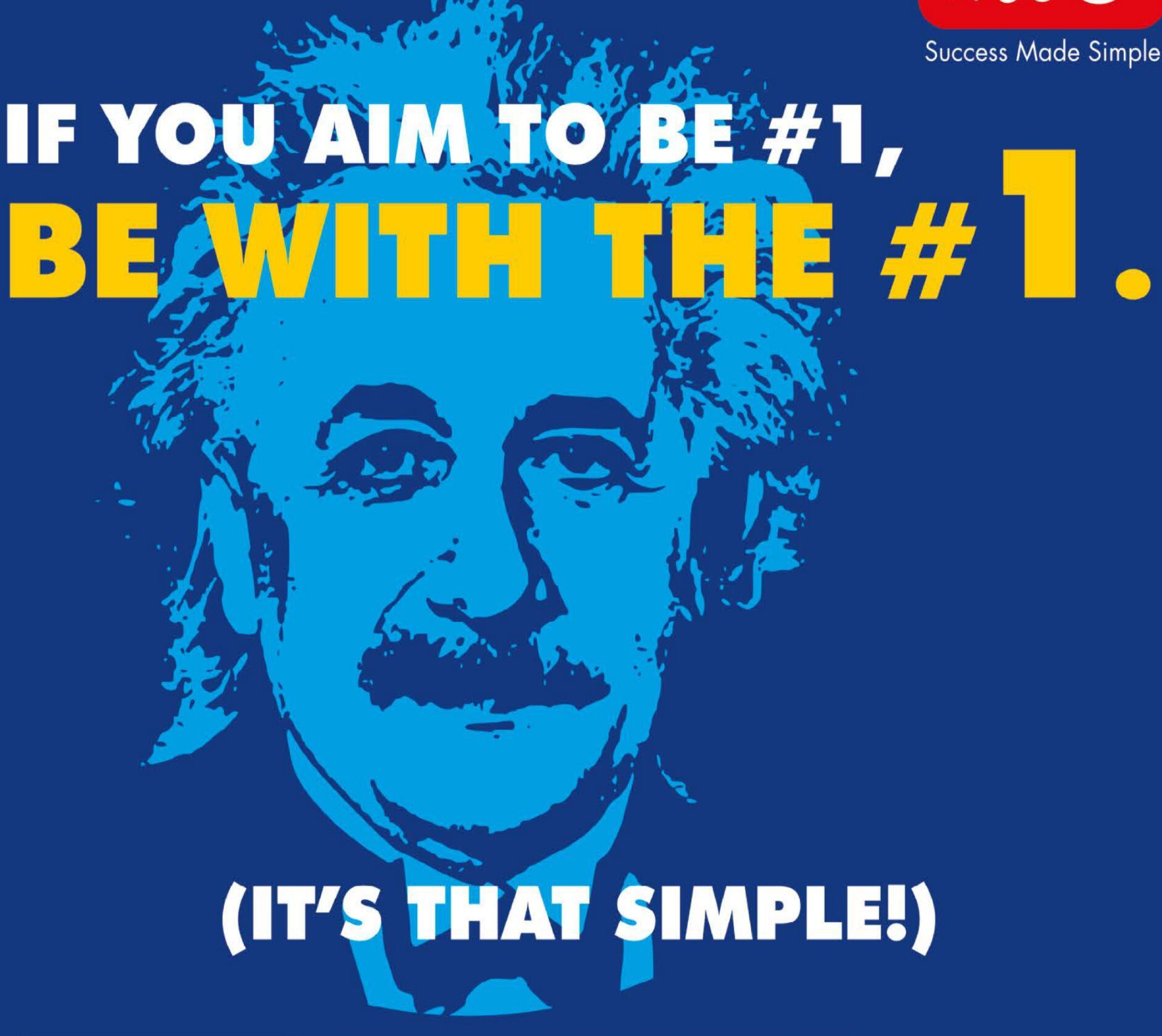
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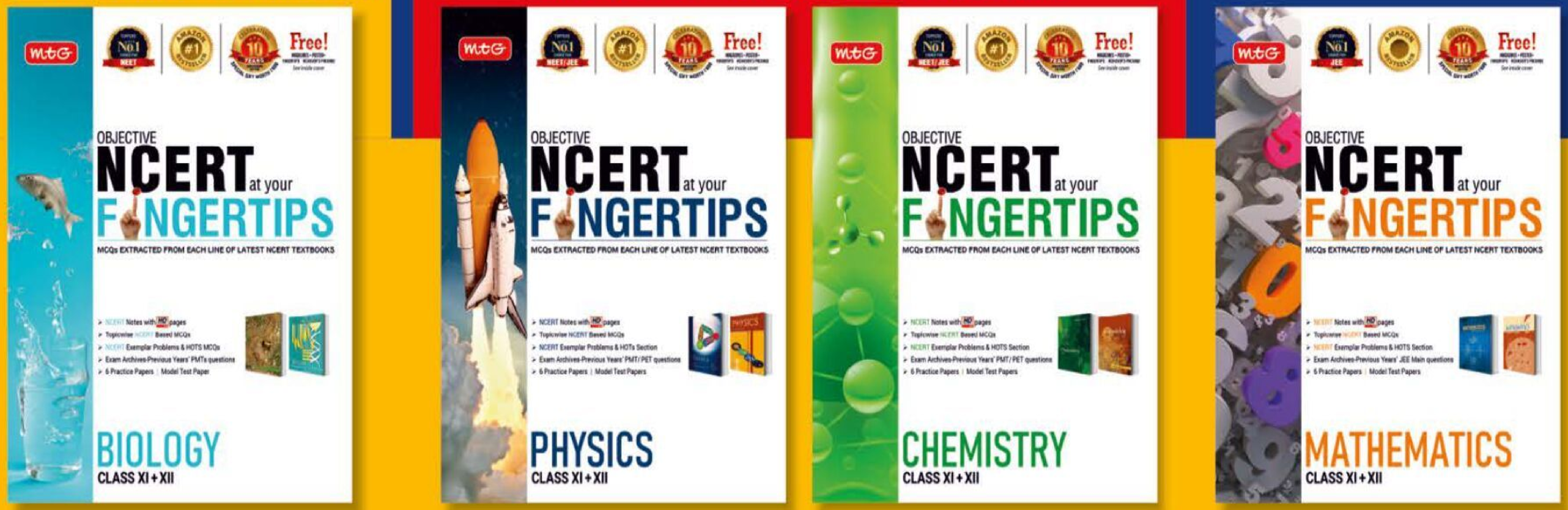
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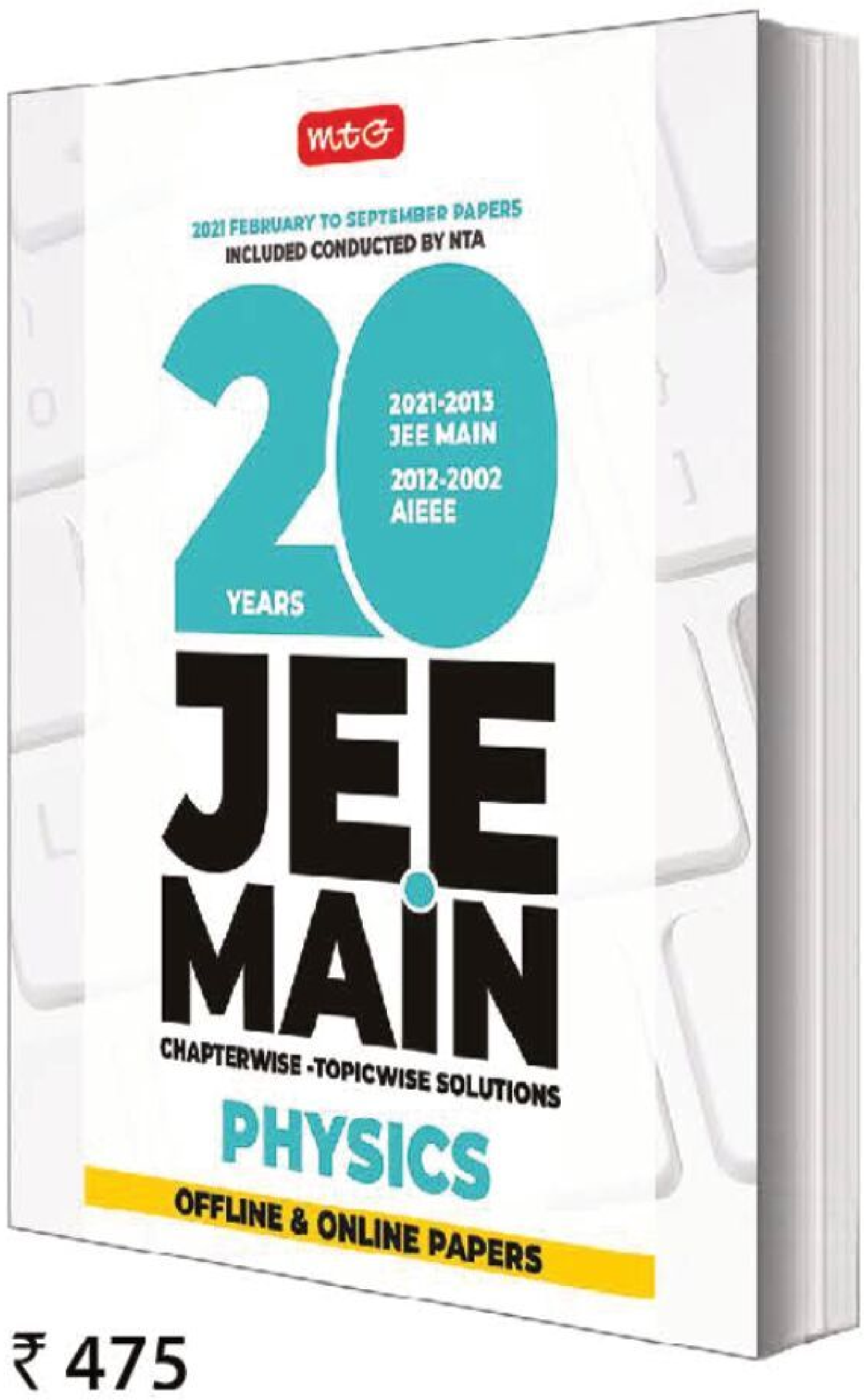
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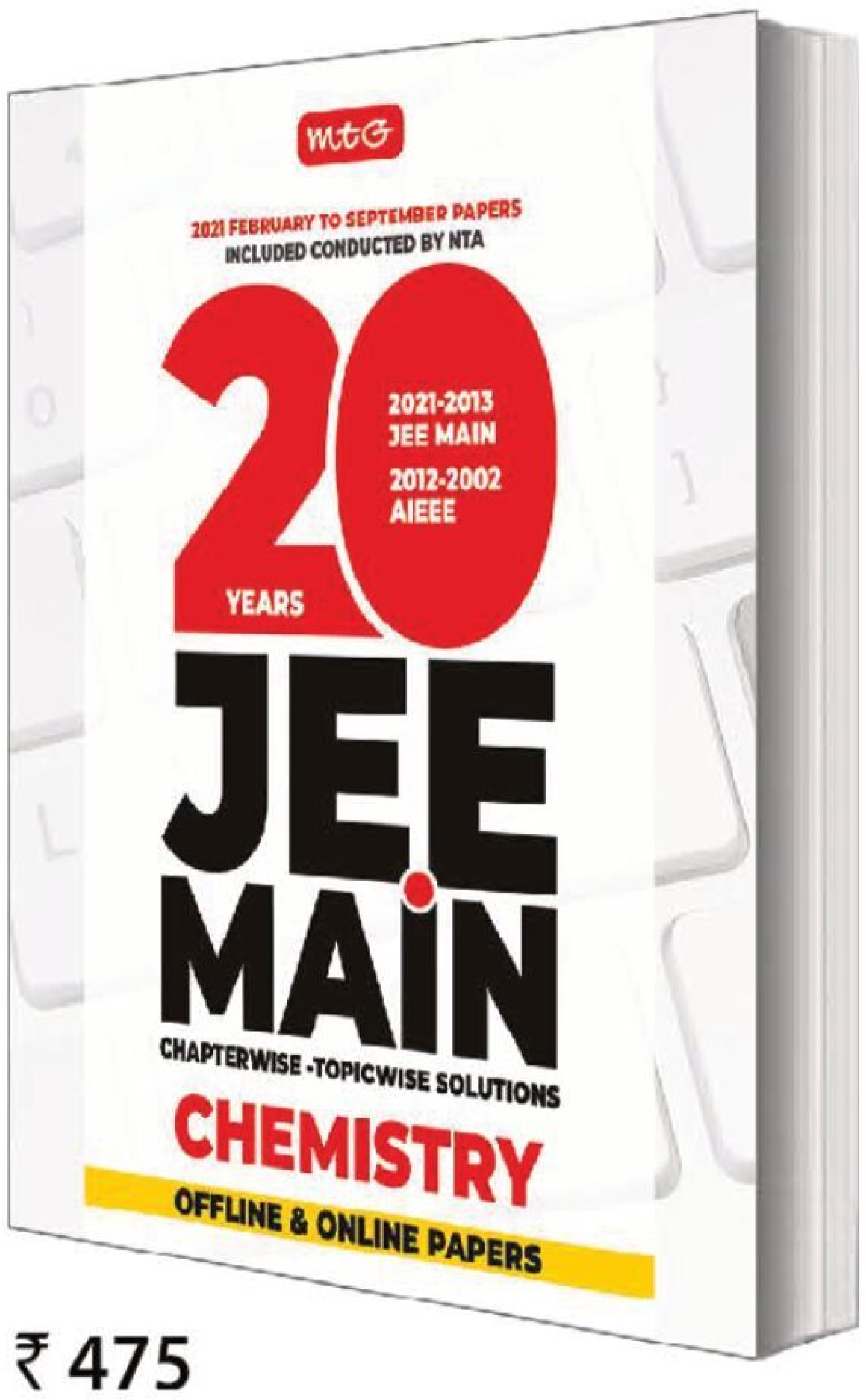
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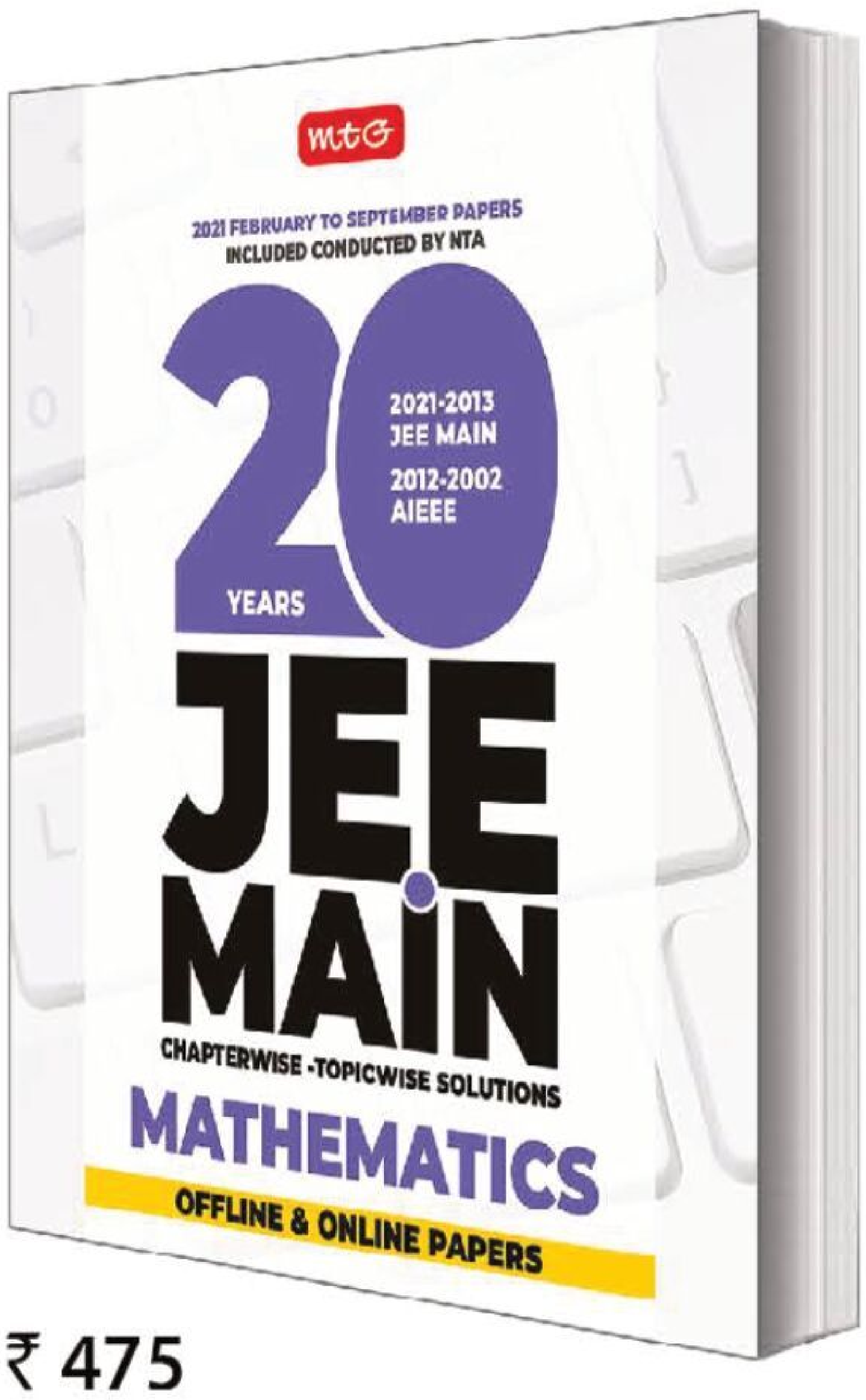
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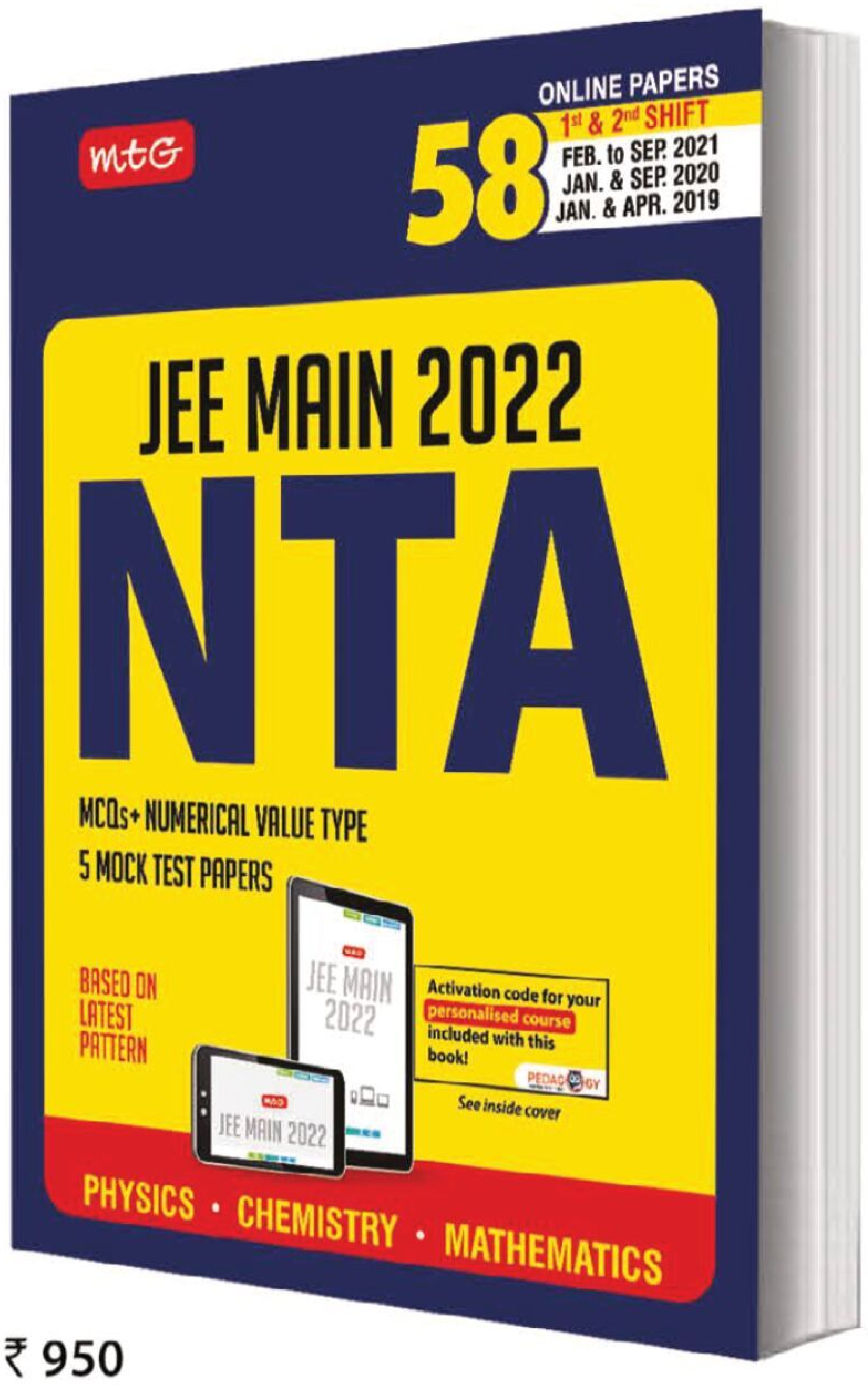
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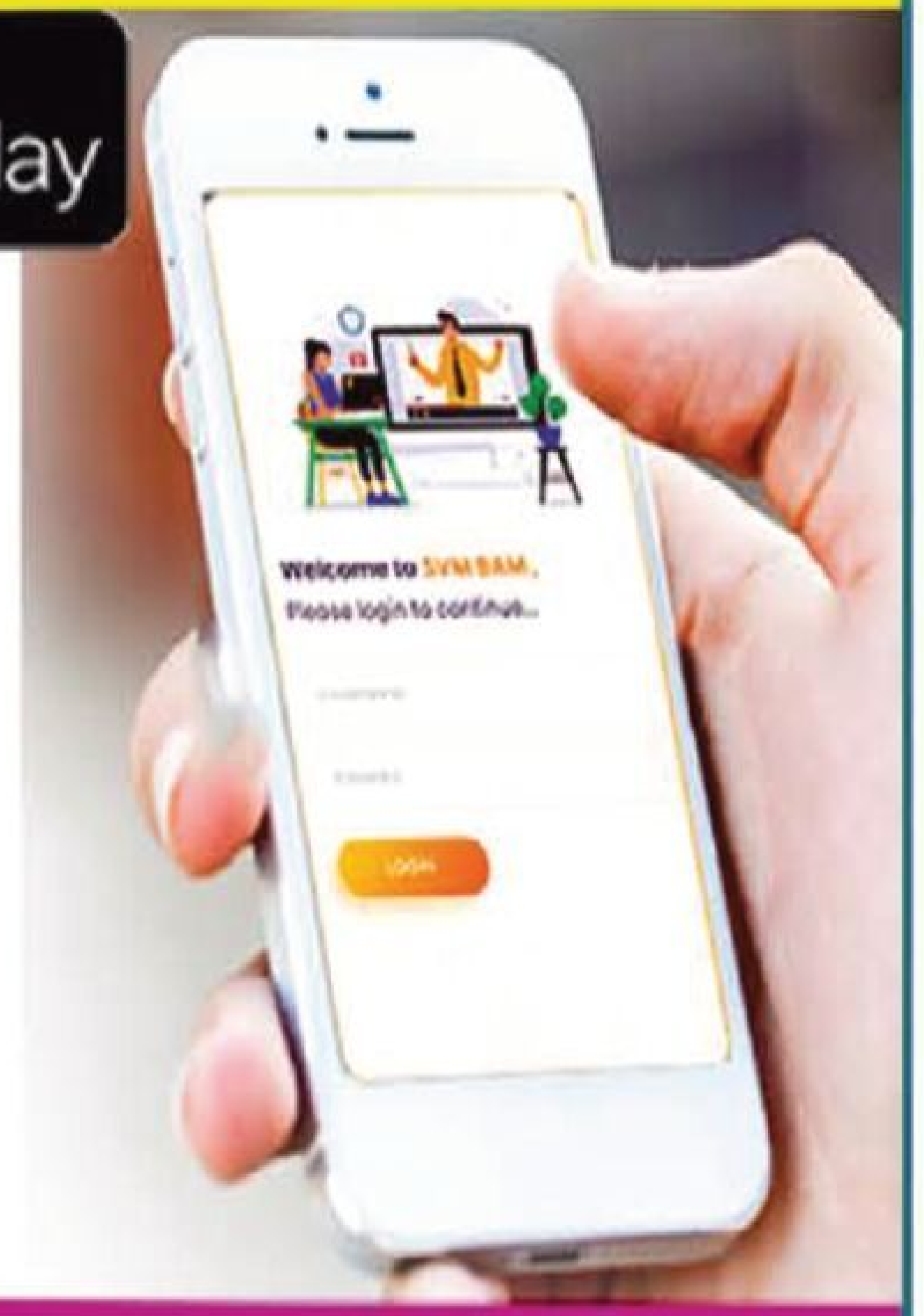
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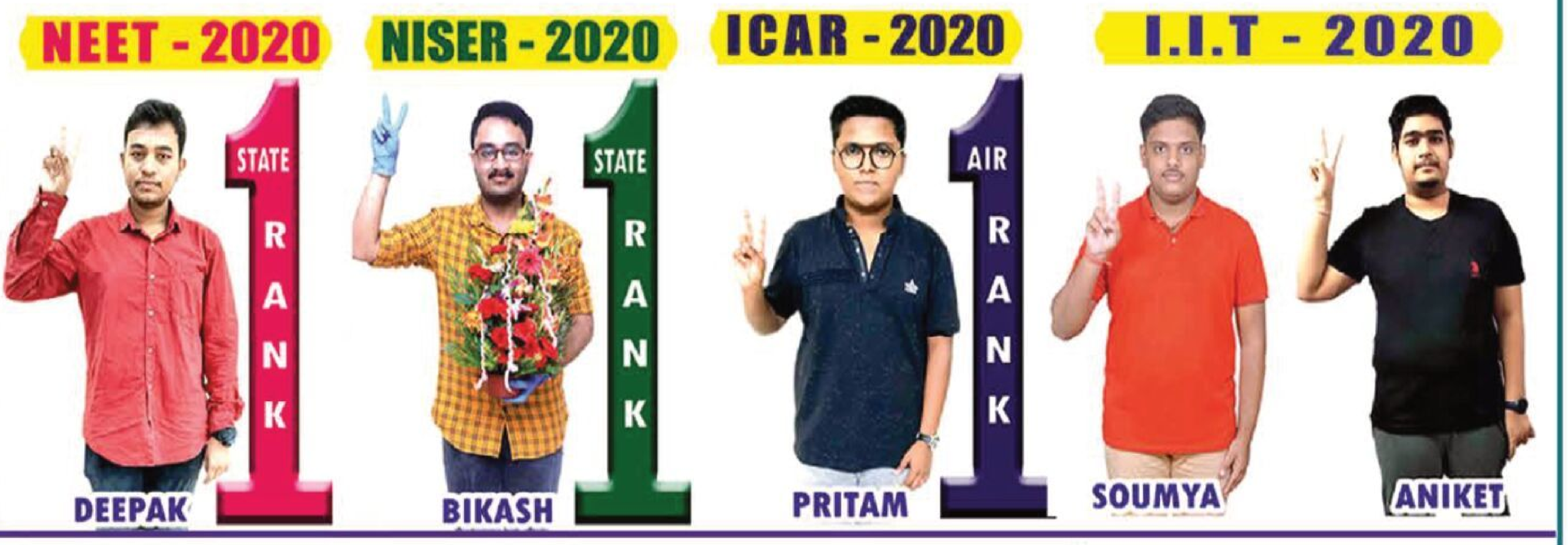
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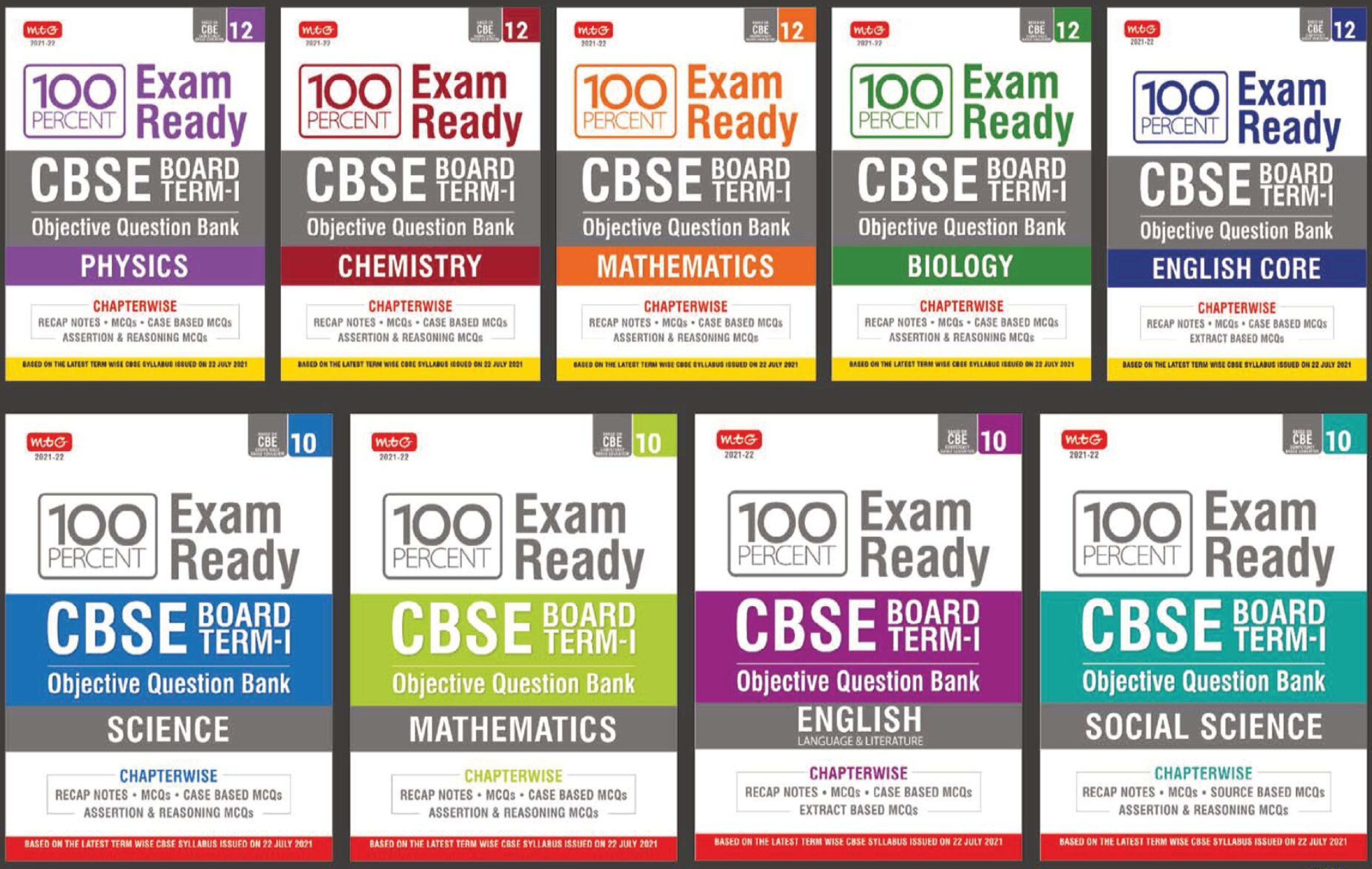
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Thinking of taking a break for JEE and NEET?

Before you decide on taking a year off to study exclusively for JEE or NEET, you need to weigh the pros and cons carefully.

To drop a year or not to, can ring in a Shakespearean dilemma for you. Taking a year's break to prepare for competitive exams such as the Joint Entrance Exam (JEE) and National Eligibility cum Entrance Test (UG) (NEET) can be a monumental decision. One needs to evaluate the pros and cons before taking a call on this. Let us help you.

Let's look at the benefits that dropping a year after your Class 12 board exams may bring.

The Pros

Shot at a better college : This may be the sole reason for taking a break for preparation, so a break can provide you the time to work hard and aim for your dream college in the next attempt.

Complete focus : This break also means you have no school or practicals to worry about. With boards out of the way, you can dedicate yourself fully to JEE or NEET preparations.

Condensed syllabus : You do not have to divert your attention to topics that may not be important for JEE or NEET. You can concentrate exclusively on the exam syllabus.

Clearing Concepts : Being clear about concepts is an important factor to do well in exams such as JEE and NEET. If you need to clear your concepts after your Class 11 and 12, dropping a year might help you.

More practice : You can devote more time to practice and go through all previous years' question papers, mock tests and chapter-wise tests that you missed before.

Working on areas to improve and time management: Taking the JEE or NEET once would have given you a good understanding about what are your strong subjects and topics. It would have also highlighted the areas you would want to improve. You can take the year to address these. You can also further improve on your time management and plan your study schedule thoroughly.

The Cons

With the extra time comes extra stress. It's possibly the first big decision you make as a young adult. Hence, you must understand what it really means and what you can expect in the coming year.

Absence of a routine : While the absence of regular school gives you more time to prepare, it also means a deviation from a daily routine. It's easy to become unproductive without supervision. If you are dropping a year, make sure to carve your own schedule and adhere to it religiously. It becomes easier if you join a coaching program and have scheduled classes. If you are struggling to follow the routine you have created, enlist parents or friends to help you stick to your schedule.

Repetitive syllabus : Picking up the same books and studying the topics you have been studying in the previous years may become tiresome. Don't forget that understanding and grasping the topics well are the reasons you are taking this break. Stay focused. Don't let lethargy set in.

Bearing boredom : With no external routines such as school or extracurriculars to follow, you may also be prone to boredom. Online games or social media can be addictive and may end up being a distraction. Cultivate healthy outlets such as outdoor sports, reading or music.

Performance pressure : Don't get bogged down by curious relatives, pressing parents or peers. Take it in your stride. Don't worry about things you cannot control. Remember: If the end result is not success, then at least you will have the satisfaction of knowing it wasn't because you didn't try.

The burden of peers : Several of your friends and schoolmates may be joining colleges and moving on to other things. Learning about their new colleges and exciting days may waver your determination and induce fear of missing out. This may cause unnecessary stress. Keep your focus on yourself. Don't lose sight of the goal.

Mental health : All the above factors may contribute to your mental stress. Falling a year behind your peers, missing out on life in college with your peers, the vast syllabus, incessant studying and the endless questions may lead you to doubt your own decision. It's natural. But you need to be strong mentally.

Know this : The long year ahead will throw up more challenges but you must tower over self-doubts to conquer what you set out to do. We hope these pros and cons help you in making a decision that's best for you. Keep your eye on the goal, everything else is a momentary distraction.

Disclaimer : The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

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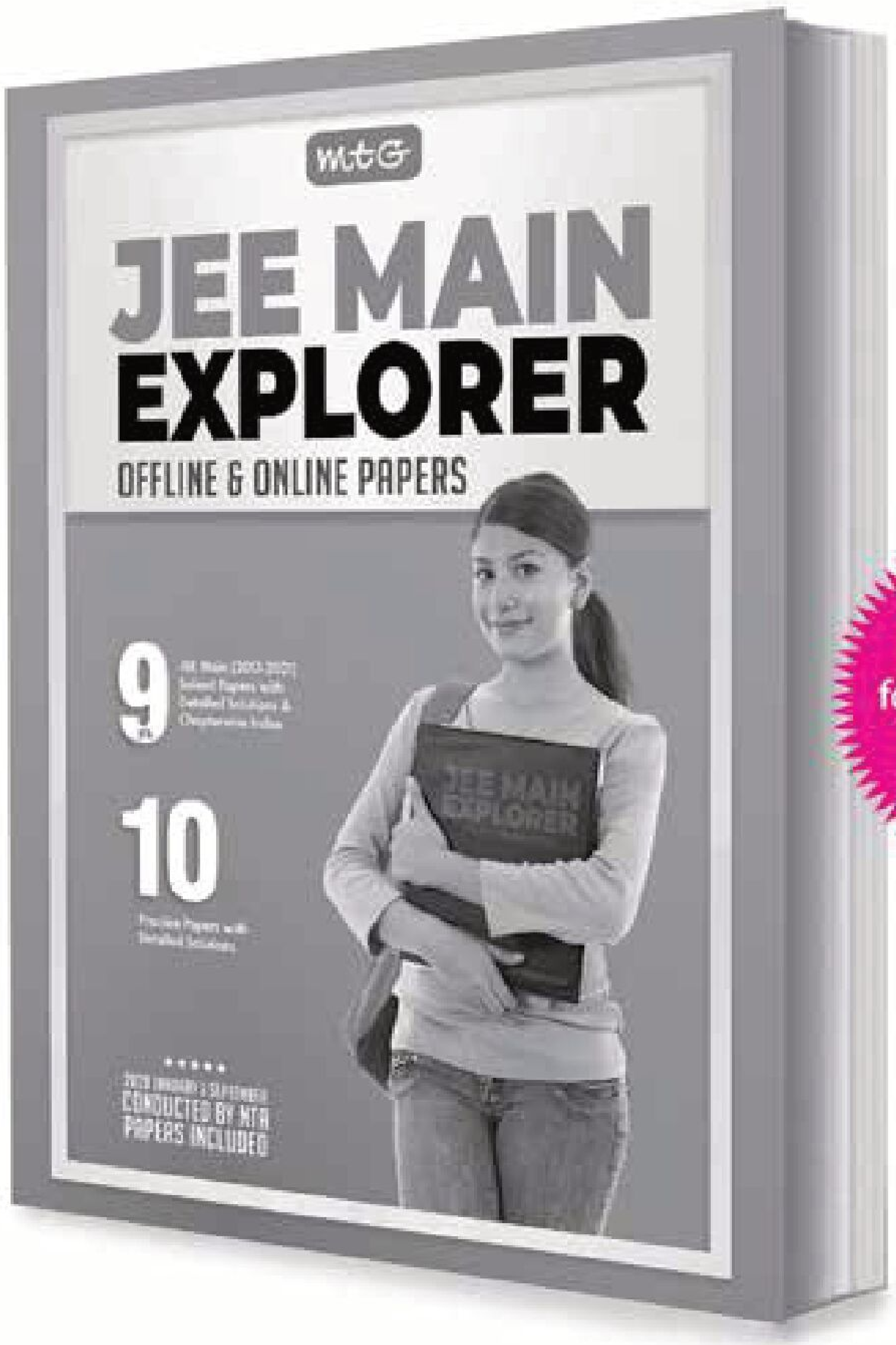
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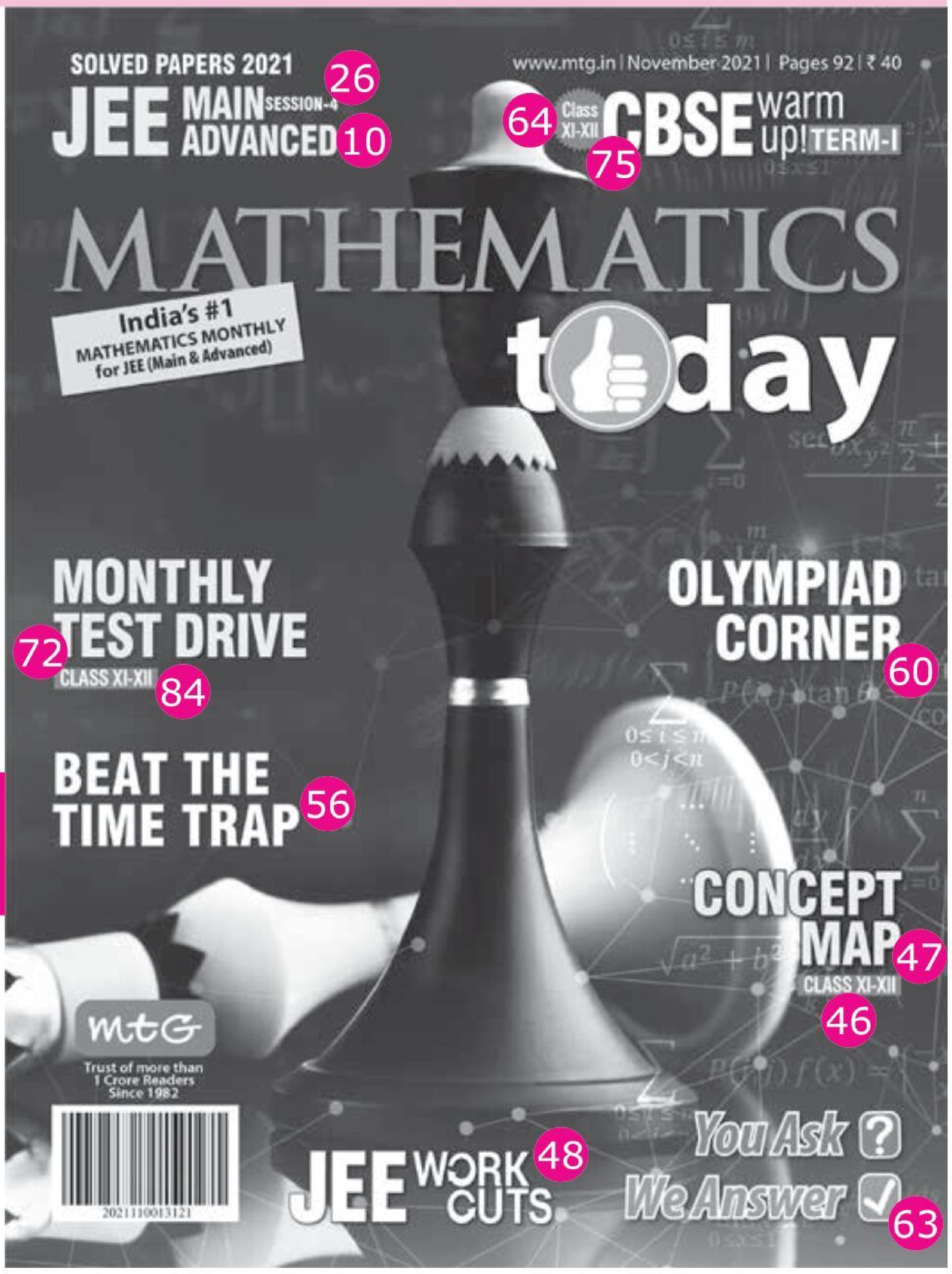
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JEE 2021
SOLVED PAPER
ADVANCED

Held on 3rd October

*Alok Kumar

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the correct option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  - *Full Marks : +3 If ONLY the correct option is chosen;*
  - *Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*
  - *Negative Marks : –1 In all other cases.*

1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(a) $x^2 + y^2 - 3x + y = 0$ (b) $x^2 + y^2 + x + 3y = 0$

(c) $x^2 + y^2 + 2y - 1 = 0$ (d) $x^2 + y^2 + x + y = 0$

2. The area of the region

$\{(x, y) : 0 \le x \le \frac{9}{4},\ 0 \le y \le 1, x \ge 3y, x + y \ge 2\}$ is

(a) $\frac{11}{32}$ (b) $\frac{35}{96}$ (c) $\frac{37}{96}$ (d) $\frac{13}{32}$

3. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$

4. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}, z_k = z_{k-1}e^{i\theta_k}$ for k = 2, 3, ..., 10, where $i = \sqrt{-1}$. Consider the statements P and Q given below :

$P : |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$

$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$

Then,

(a) P is TRUE and Q is FALSE

(b) Q is TRUE and P is FALSE

(c) both P and Q are TRUE

(d) both P and Q are FALSE

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.*
- *If the numerical value has more than two decimal places, truncate/round-off the value of TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme :*
  - *Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;*
  - *Zero Marks : 0 In all other cases.*

Question Stem for Question Nos. 5 and 6

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

5. The value of $\frac{625}{4} p_1$ is _______.

---

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

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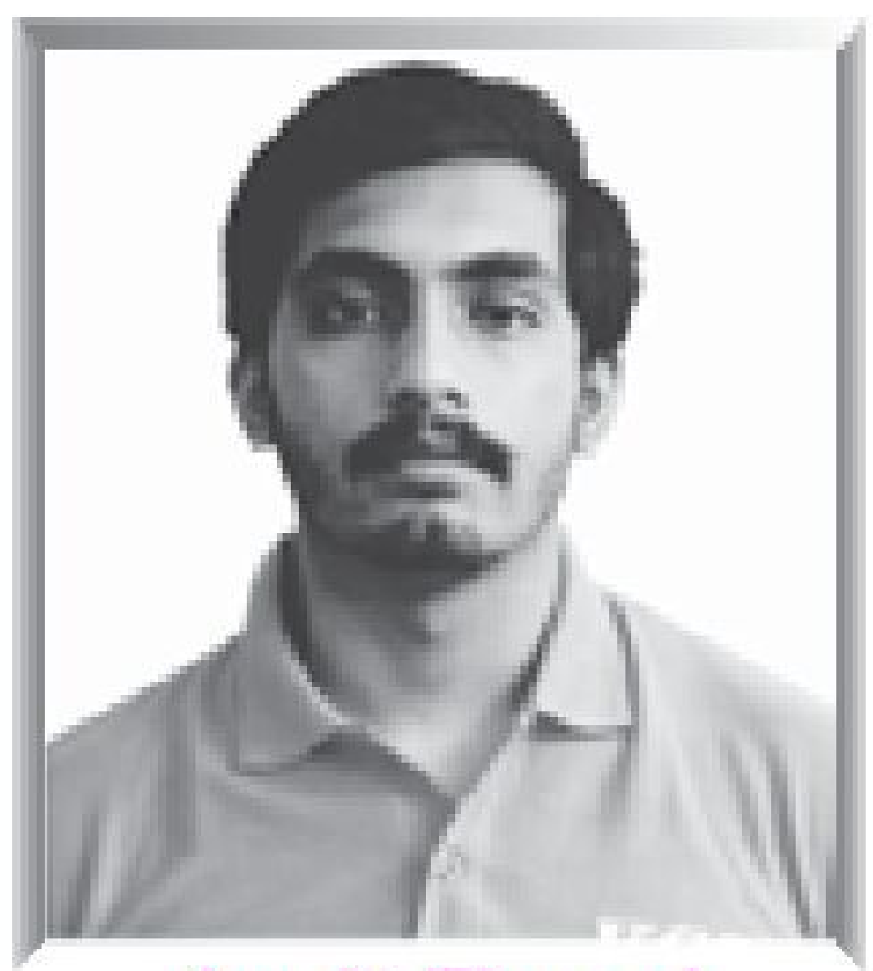

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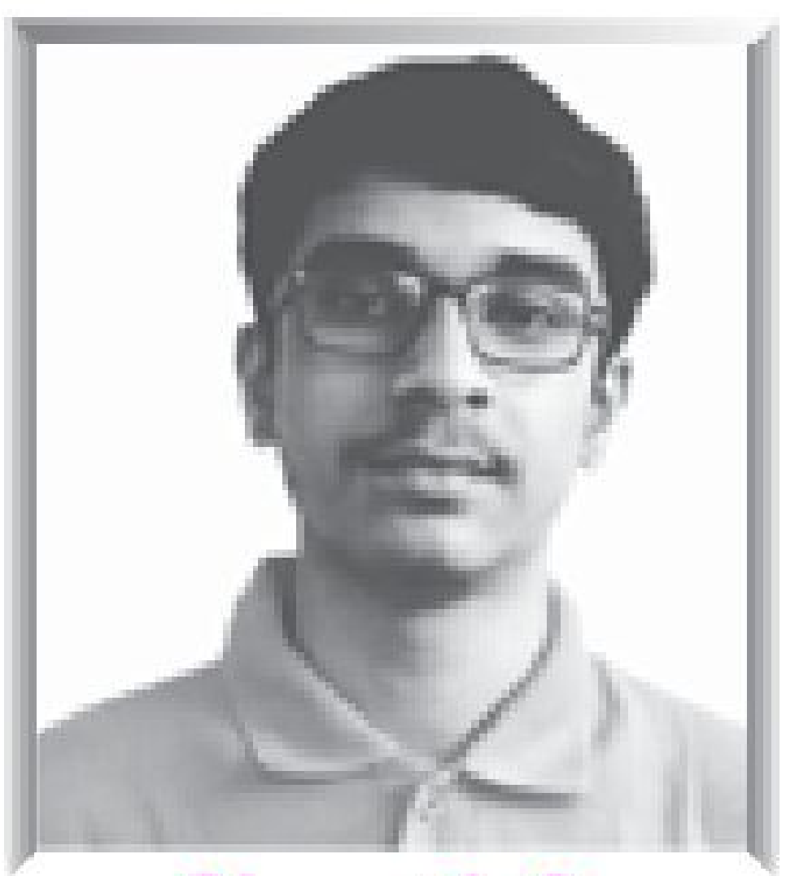

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6. The value of $\frac{125}{4}p_2$ is ________.

Question Stem for Question Nos. 7 and 8

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

7. The value of $|M|$ is ________.

8. The value of D is ________.

Question Stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$$L_1 : x\sqrt{2} + y - 1 = 0 \text{ and } L_2 : x\sqrt{2} - y + 1 = 0$$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

9. The value of λ^2 is _______.

10. The value of D is _______.

SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks : +4 *If only (all) the correct option(s) is (are) chosen;*
Partial Marks : + 3 *If all the four options are correct but ONLY three options are chosen;*
Partial Marks : + 2 *If three or more options are correct but ONLY two options are chosen, both of which are correct;*
Partial Marks : + 1 *If two or more options are correct but ONLY one option is chosen and it is a correct option;*
Zero Marks : 0 *If unanswered;*
Negative Marks : –2 *In all other cases.*

- *For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then*
  choosing ONLY (a), (b) and (d) will get +4 marks;
  choosing ONLY (a) and (b) will get +2 marks;
  choosing ONLY (a) and (d) will get +2 marks;
  choosing ONLY (b) and (d) will get +2 marks;
  choosing ONLY (a) will get +1 marks;
  choosing ONLY (b) will get +1 marks;
  choosing ONLY (d) will get +1 marks;
- *choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.*

11. For any 3 × 3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3 × 3, then which of the following statements is (are) TRUE?

(a) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(c) $|(EF)^3| > |EF|^2$

(d) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

12. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) TRUE?

(a) f is decreasing in the interval $(-2, -1)$

(b) f is increasing in the interval $(1, 2)$

(c) f is onto

(d) Range of f is $\left[-\frac{3}{2}, 2\right]$

13. Let E, F and G be three events having probabilities $P(E)=\frac{1}{8},\ P(F)=\frac{1}{6}$ and $P(G)=\frac{1}{4}$,

and let $P(E\cap F\cap G)=\frac{1}{10}$.

For any event H, if H^c denotes its complement, then which of the following statements is (are) TRUE?

(a) $P(E\cap F\cap G^c)\le\frac{1}{40}$ (b) $P(E^c\cap F\cap G)\le\frac{1}{15}$

(c) $P(E\cup F\cup G)\le\frac{13}{24}$ (d) $P(E^c\cap F^c\cap G^c)\le\frac{5}{12}$

14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I-EF)$ is invertible. If $G=(I-EF)^{-1}$, then which of the following statements is (are) TRUE?

(a) $|FE|=|I-FE|\,|FGE|$ (b) $(I-FE)(I+FGE)=I$

(c) $EFG=GEF$ (d) $(I-FE)(I-FGE)=I$

15. For any positive integer n, let $S_n:(0,\infty)\to\mathbb{R}$ be defined by

$$S_n(x)=\sum_{k=1}^{n}\cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x\in\mathbb{R}$, $\cot^{-1}(x)$ $(0,\pi)$ and $\tan^{-1}(x)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE?

(a) $S_{10}(x)=\frac{\pi}{2}-\tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x>0$

(b) $\lim\limits_{n\to\infty}\cot(S_n(x))=x$, for all $x>0$

(c) The equation $S_3(x)=\frac{\pi}{4}$ has a root in $(0,\infty)$

(d) $\tan(S_n(x))\le\frac{1}{2}$, for all $n\ge 1$ and $x>0$

16. For any complex number $w=c+id$, let $\arg(w)\in(-\pi,\pi]$, where $i=\sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z=x+iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right)=\frac{\pi}{4}$, the ordered pair (x,y) lies on the circle $x^2+y^2+5x-3y+4=0$

Then which of the following statements is (are) TRUE?

(a) $\alpha=-1$ (b) $\alpha\beta=4$

(c) $\alpha\beta=-4$ (d) $\beta=4$

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks :	*+4*	*If ONLY the correct integer is entered;*
Zero Marks :	*0*	*In all other cases.*

17. For $x\in\mathbb{R}$, the number of real roots of the equation $3x^2-4|x^2-1|+x-1=0$ is ________.

18. In a triangle ABC, let $AB=\sqrt{23}$, $BC=3$ and $CA=4$. Then the value of $\frac{\cot A+\cot C}{\cot B}$ is ________.

19. Let $\vec{u},\vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u}\cdot\vec{w}=1$, $\vec{v}\cdot\vec{w}=1$, $\vec{w}\cdot\vec{w}=4$. If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u},\vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u}+5\vec{v}|$ is ________.

PAPER-2

SECTION 1

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks :	*+4*	*If only (all) the correct option(s) is (are) chosen;*
Partial Marks :	*+ 3*	*If all the four options are correct but ONLY three options are chosen;*
Partial Marks :	*+ 2*	*If three or more options are correct but ONLY two options are chosen, both of which are correct;*
Partial Marks :	*+ 1*	*If two or more options are correct but ONLY one option is chosen and it is a correct option;*
Zero Marks :	*0*	*If unanswered;*
Negative Marks :	*–2*	*In all other cases;*

- *For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then*

choosing ONLY (a), (b) and (d) will get +4 marks;
choosing ONLY (a) and (b) will get +2 marks;
choosing ONLY (a) and (d) will get +2 marks;
choosing ONLY (a) and (d) will get +2 marks;
choosing ONLY (a) will get +1 marks;
choosing ONLY (b) will get +1 marks;
choosing ONLY (d) will get +1 marks;

- ***choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.***

1. Let $S_1 = \{(i, j, k) : i, j, k \in \{1, 2, ..., 10\}\}$,
$S_2 = \{(i, j) : 1 \le i < j + 2 \le 10,\ i, j \in \{1, 2, ..., 10\}\}$,
$S_3 = \{(i, j, k, l) : 1 \le i < j < k < l,\ i, j, k, l \in \{1, 2, ..., 10\}\}$ and
$S_4 = \{(i, j, k, l) : i, j, k$ and l are distinct elements in $\{1, 2, ..., 10\}\}$.
If the total number of elements in the set S_r is n_r, $r = 1, 2, 3, 4$, then which of the following statements is (are) TRUE?

(a) $n_1 = 1000$ (b) $n_2 = 44$

(c) $n_3 = 220$ (d) $\frac{n_4}{12} = 420$

2. Consider a triangle PQR having sides of lengths p, q and r opposite to the angles P, Q and R, respectively. Then which of the following statements is (are) TRUE?

(a) $\cos P \ge 1 - \frac{p^2}{2qr}$

(b) $\cos R \ge \left(\frac{q-r}{p+q}\right)\cos P + \left(\frac{p-r}{p+q}\right)\cos Q$

(c) $\frac{q+r}{p} < 2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

(d) If $p < q$ and $p < r$, then $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

3. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)dt = 0$

Then which of the following statements is (are) TRUE?

(a) The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(b) The equation $f(x) - 3\sin 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(c) $\lim\limits_{x\to 0} \frac{x\int_0^x f(t)dt}{1 - e^{x^2}} = -1$

(d) $\lim\limits_{x\to 0} \frac{\sin x\int_0^x f(t)dt}{x^2} = -1$

4. For any real numbers α and β, let $y_{\alpha,\beta}(x)$, $x \in \mathbb{R}$, be the solution of the differential equation

$\frac{dy}{dx} + \alpha y = xe^{\beta x}$, $y(1) = 1$.

Let $S = \{y_{\alpha,\beta}(x) : \alpha, \beta \in \mathbb{R}\}$. Then which of the following functions belong(s) to the set S?

(a) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

(b) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

(c) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

(d) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

5. Let O be the origin and $\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OB} - \lambda\overrightarrow{OA})$ for some $\lambda > 0$. If $|\overrightarrow{OB} \times \overrightarrow{OC}| = \frac{9}{2}$, then which of the following statements is (are) TRUE?

(a) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(b) Area of the triangle OAB is $\frac{9}{2}$

(c) Area of the triangle ABC is $\frac{9}{2}$

(d) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

6. Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE?

(a) The triangle PFQ is a right-angled triangle
(b) The triangle QPQ' is a right-angled triangle
(c) The distance between P and F is $5\sqrt{2}$
(d) F lies on the line joining Q and Q'

SECTION 2

- ***This section contains THREE (03) question stems.***
- ***There are TWO (02) questions corresponding to each question stem.***

18+

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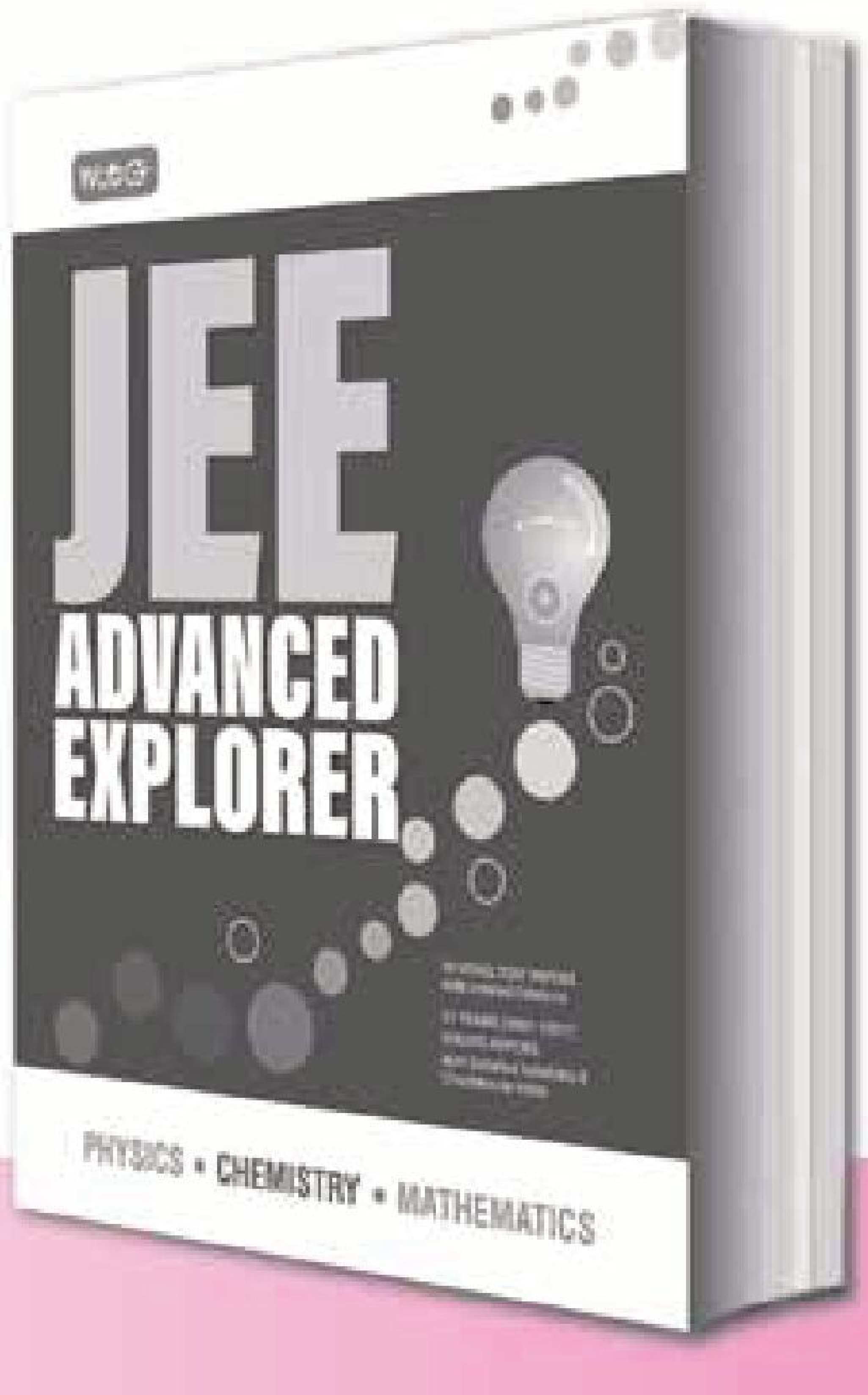
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- ***The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.***
- ***If the numerical value has more than two decimal places, truncate/round-off the value of TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme :***
  Full Marks : +2 If only the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

Consider the region $R = \{x, y\} \in \mathbb{R}\times\mathbb{R} : x \geq 0$ and $y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

7. The radius of the circle C is ________.

8. The value of α is ________.

Question Stem for Question Nos. 9 and 10

Question Stem

Let $f_1 : (0, \infty) \to \mathbb{R}$ and $f_2 : (0, \infty) \to \mathbb{R}$ be defined by

$$f_1(x) = \int_0^x \prod_{j=1}^{21} (t - j)^j \, dt, \; x > 0$$

and $f_2(x) = 98(x - 1)^{50} - 600(x - 1)^{49} + 2450$, $x > 0$, where, for any positive integer n and real numbers $a_1, a_2, \ldots\ldots, a_n$, $\prod_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots\ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, $i = 1, 2$, in the interval $(0, \infty)$.

9. The value of $2m_1 + 3n_1 + m_1n_1$ is ________.

10. The value of $6m_2 + 4n_2 + 8m_2n_2$ is ________.

Question Stem for Question Nos. 11 and 12

Question Stem

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}, i = 1, 2$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$ be functions such that $g_1(x) = 1$, $g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x)\cdot g_i(x)\,dx, \; i = 1, 2$

11. The value of $\frac{16S_1}{\pi}$ is ________.

12. The value of $\frac{48S_2}{\pi^2}$ is ________.

SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***
- ***Answer to each question will be evaluated according to the following marking scheme :***
  Full Marks : +3 If only the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Paragraph Q. 13-14

Let $M = \{(x, y) \in \mathbb{R}\times\mathbb{R} : x^2 + y^2 \leq r^2\}$, where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, $n = 1, 2, 3, \ldots\ldots$. Let $S_0 = 0$ and for $n \geq 1$, let S_n denote the sum of the first n terms of this progression. For $n \geq 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

13. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then
(a) $k + 2l = 22$ (b) $2k + l = 26$
(c) $2k + 3l = 34$ (d) $3k + 2l = 40$

14. Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is
(a) 198 (b) 199 (c) 200 (d) 201

Paragraph Q. 15-16

Let $\psi_1 : [0, \infty) \to \mathbb{R}$, $\psi_2 : [0, \infty] \to \mathbb{R}$, $f : [0, \infty) \to \mathbb{R}$, and $g : [0, \infty] \to \mathbb{R}$, be functions such that $f(0) = g(0) = 0$,

$$\psi_1(x) = e^{-x} + x, \; x \geq 0$$
$$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, \; x \geq 0$$
$$f(x) = \int_{-x}^{x} (|t| - t^2)\, e^{-t^2}\, dt, \; x > 0.$$

and $g(x) = \int_0^{x^2} \sqrt{t}\, e^{-t}\, dt, \; x > 0$

15. Which of the following statements is TRUE?
(a) $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

(b) For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

(c) For every $x > 0$, there exists a $\beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

(d) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

16. Which of the following statements is TRUE?

(a) $\psi_1(x) \le 1$, for all $x > 0$

(b) $\psi_2(x) \le 0$, for all $x > 0$

(c) $f(x) \ge 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(d) $g(x) \le \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks : +4 *If ONLY the correct integer is entered;*
Zero Marks : 0 *In all other cases.*

17. A number is chosen at random from the set {1, 2, 3, ..., 2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of $500p$ is ______.

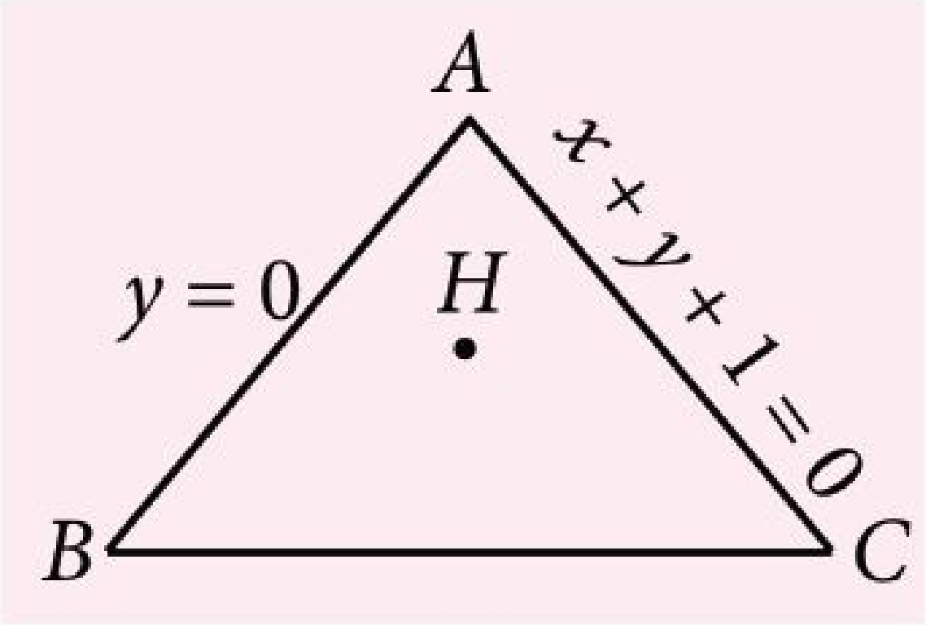
18. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. For any three distinct points P, Q and Q' on E, let $M(P, Q)$ be the mid-point of the line segment joining P and Q, and $M(P, Q')$ be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as P, Q and Q' vary on E, is ______.

19. For any real number x, let $[x]$ denote the largest integer less than or equal to x. If $I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$, then the value of $9I$ is ______.

SOLUTIONS

PAPER-1

1. **(b) : 1st solution :**



Vertex A is given by $(-1, 0)$. Let H be orthocentre = $(1, 1)$.
Let $B(q, 0)$ and $C(r, -r-1)$

$\because \; BH \perp AC \; \therefore \; \frac{1}{1-q} \cdot (-1) = -1 \Rightarrow 1 = 1 - q$

$\Rightarrow \; q = 0$

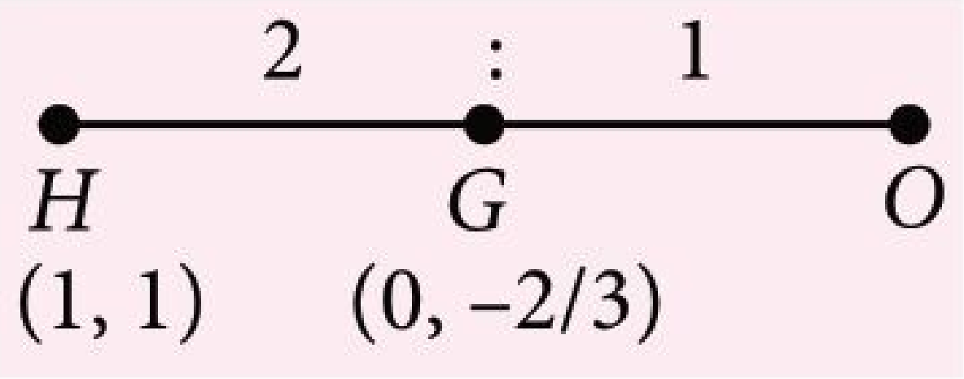
Thus, B is $(0, 0)$.

Similarly, $CH \perp AB \Rightarrow CH$ is parallel to x-axis.

$\Rightarrow \; r = 1$ and so, C is $(1, -2)$

Centroid of ΔABC (G) is $(0, -2/3)$

Circumcentre, O is given by



$\left(-\frac{1}{2}, -\frac{3}{2}\right)$ [By section formula]

Thus, the circumcircle is $x^2 + y^2 + x + 3y = 0$.

2nd solution: The reflection of orthocentre about any side of triangle lies on the circumcircle.

Image of $(1, 1)$ about $y = 0$ is $(1, -1) \equiv P$

And image of $(1, 1)$ about $x + y + 1 = 0$ is $(-2, -2) \equiv Q$ and $A(-1, 0)$ is the vertex.

As the circumcircle passes through P, Q and A, so its equation can be assumed to be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Since, P, A, Q lie on the above locus, so

$$2 + 2g - 2f + c = 0 \;\text{...(i)}\quad 1 - 2g + c = 0 \;\text{...(ii)}$$

and $8 - 4g - 4f + c = 0$ ...(iii)

Solving (i), (ii) and (iii), we get $g = \frac{1}{2}, \; f = \frac{3}{2}, \; c = 0$

$\therefore$ Required equation is $x^2 + y^2 + x + 3y = 0$

[Topic : Circle]

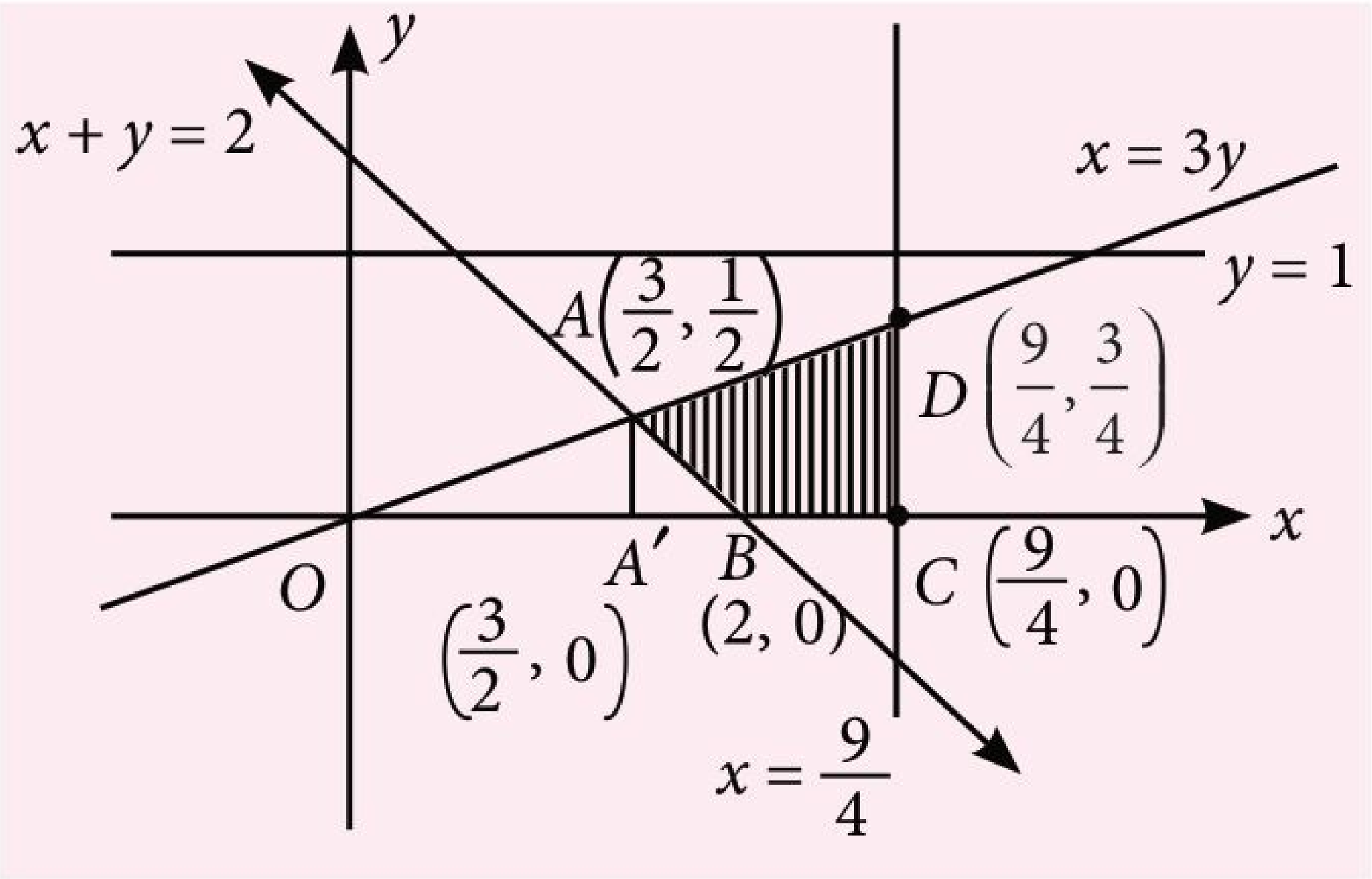
[Rating : Easy]

2. **(a) :** $0 \le x \le \frac{9}{4}$ and $0 \le y \le 1$ determine a rectangle.

Now, solve the equation $x = 3y$ and $x + y = 2$ to find the point of intersection to determine the boundary.

$\therefore$ Area of $ABCD$ = Area of $AA'CD$ – Area of $AA'B$

$$= \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{3}{4}\right) \cdot \left(\frac{9}{4} - \frac{3}{2}\right) - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{5}{4} \cdot \frac{3}{4} - \frac{1}{8} = \frac{11}{32}$$


y
x + y = 2
x = 3y
y = 1
$A\left(\frac{3}{2},\frac{1}{2}\right)$
$D\left(\frac{9}{4},\frac{3}{4}\right)$
x
O
A′
B
$C\left(\frac{9}{4},0\right)$
$\left(\frac{3}{2},0\right)$
(2, 0)
$x=\frac{9}{4}$

[Topic : Area bounded by curves]
[Rating : Easy]

3. (a) : Let $P(X_{12})$ be the probability of choosing 1, 2 from E_1 in the beginning.
Analogously define $P(X_{23})$, $P(X_{31})$
Let $P(X) = P(X_{12}) + P(X_{23}) + P(X_{31})$

$$p=\frac{P(S_1\cap(E_1=E_3))}{P(E_1=E_3)}=\frac{P(X_{12})}{P(X)}$$

$$\text{Now, } P(X_{12})=\frac{1}{3}\cdot\frac{1\cdot {}^3C_1}{{}^4C_2}\cdot\frac{1}{{}^5C_2}=\frac{1}{60}$$

$$P(X_{23})=\frac{1}{3}\cdot\left\{\frac{1\cdot {}^3C_2}{{}^4C_2}\cdot\frac{1}{{}^4C_2}+\frac{1\cdot {}^3C_1}{{}^4C_2}\cdot\frac{1}{{}^5C_2}\right\}$$

$$=\frac{1}{3}\left(\frac{1}{12}+\frac{1}{20}\right)=\frac{1}{3}\cdot\frac{8}{60}=\frac{2}{45}$$

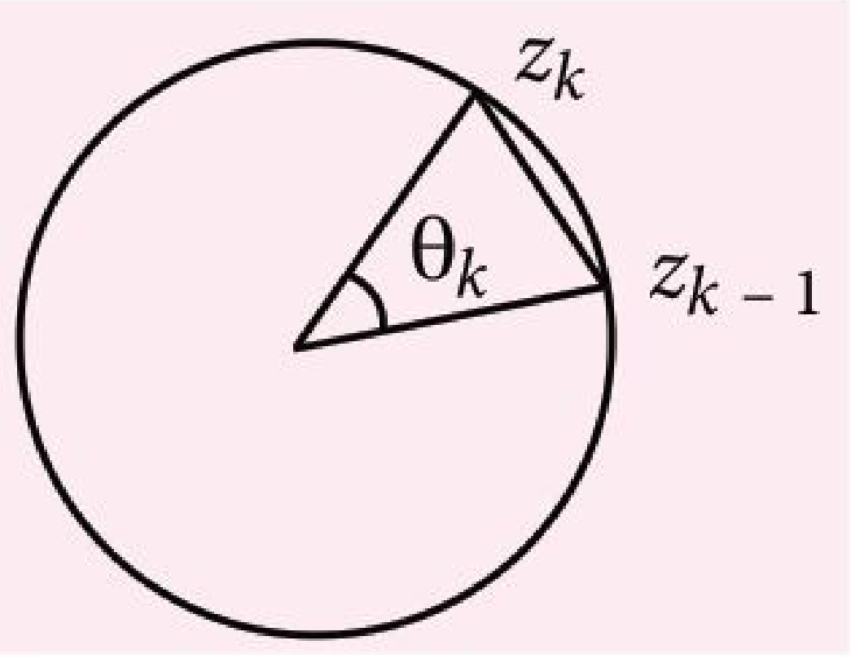
$$P(X_{31})=\frac{1}{3}\cdot\frac{1\cdot {}^2C_1}{{}^3C_2}\cdot\frac{1}{{}^5C_2}=\frac{1}{45}$$

$$\text{So, } p=\frac{\frac{1}{60}}{\frac{1}{60}+\frac{2}{45}+\frac{1}{45}}=\frac{\frac{1}{60}}{\frac{3+8+4}{180}}=\frac{1}{60}\cdot\frac{180}{15}=\frac{1}{5}$$

[Topic : Probability]
[Rating : Difficult]

4. (c) : We have, $|z_2 - z_1| \le \theta_1$
$|z_3 - z_2| \le \theta_2$

$|z_1 - z_{10}| \le \theta_{10}$



Adding all these, we get
$|z_2 - z_1| + |z_3 - z_2| + \ldots + |z_1 - z_{10}| \le 2\pi$
Now, $|z_2^2 - z_1^2| = |z_2 - z_1|\,|z_2 + z_1| \le 2|z_2 - z_1|$
$[\because\ |z_2 + z_1| \le |z_2| + |z_1| = 2]$

We have, $|z_2^2-z_1^2|+|z_3^2-z_2^2|+\ldots+|z_1^2-z_{10}^2|\le 2\cdot 2\pi=4\pi$

[Topic : Complex Numbers]
[Rating : Medium]

5. (76.25) : $p_1 = 1 -$ Probability (all 3 numbers ≤ 80)

$$=1-\left(\frac{80}{100}\right)^3=1-\left(\frac{4}{5}\right)^3=\frac{61}{125}$$

$$\therefore\ \frac{625}{4}p_1=\frac{625}{4}\times\frac{61}{125}=76.25$$

6. (24.5) : $p_2 = 1 -$ Probability (all 3 numbers are ≥ 41)

$$=1-\left(\frac{60}{100}\right)^3=1-\frac{27}{125}=\frac{98}{125}$$

$$\therefore\ \frac{125}{4}p_2=\frac{125}{4}\cdot\frac{98}{125}=\frac{98}{4}=24.5$$

[Topic : Probability]
[Rating : Medium]

(7-8) : $x + 2y + 3z = \alpha$ ...(i)
$4x + 5y + 6z = \beta$ ...(ii)
$7x + 8y + 9z = \gamma - 1$ ...(iii)
Adding (i) and (iii) and subtracting twice of (ii) gives
$\alpha + \gamma - 1 - 2\beta = 0$
$\Rightarrow\ \alpha - 2\beta + \gamma - 1 = 0$ ...(iv)

$$\text{Now, } |M|=\begin{vmatrix}\alpha & 2 & \gamma\\ \beta & 1 & 0\\ -1 & 0 & 1\end{vmatrix}=\alpha-2\beta+\gamma=1 \qquad \text{[Using (iv)]}$$

Thus, the plane P is $x - 2y + z = 1$
The perpendicular distance of (0, 1, 0) from the plane P

$$\text{is } \left|\frac{-3}{\sqrt{1^2+2^2+1^2}}\right|=\frac{3}{\sqrt{6}}=\sqrt{\frac{9}{6}}=\sqrt{\frac{3}{2}}$$

$$\therefore\ D=\left(\sqrt{\frac{3}{2}}\right)^2=\frac{3}{2}=1.5$$

7. (1) **8. (1.5)**

[Topic : Linear Equations]
[Rating : Easy]

9. (9.00) : The locus of P is given by

$$\left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\cdot\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right|=\lambda^2$$

$$\Rightarrow\ \left(\frac{2x^2-(y-1)^2}{3}\right)=\lambda^2$$

$\Rightarrow\ C : |2x^2 - (y - 1)^2| = 3\lambda^2$ ...(i)
Line : $y = 2x + 1$ meets C at $R(x_1, y_1)$ and $S(x_2, y_2)$.

$$\text{Now, } RS=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$=\sqrt{(x_1-x_2)^2+4(x_1-x_2)^2}$

[∵ $y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$ ∴ $y_1 - y_2 = 2(x_1 - x_2)$]

$=\sqrt{5}\,|x_1-x_2|$

$y = 2x + 1$ meets the curve C

∴ $|2x^2 - (2x + 1 - 1)^2| = 3\lambda^2$

∴ $|2x^2| = 3\lambda^2 \Rightarrow x^2=\frac{3}{2}\lambda^2$

∴ $x_1=\sqrt{\frac{3}{2}}\,|\lambda|,\ x_2=-\sqrt{\frac{3}{2}}\,|\lambda|$

We have, $RS=\sqrt{5}\,|x_1-x_2|=\sqrt{5}\cdot 2\sqrt{\frac{3}{2}}\,|\lambda|=\sqrt{270}$

$\Rightarrow\ 5\cdot 4\cdot\frac{3}{2}\lambda^2=270\ \Rightarrow\ \lambda^2=9$

10. (77.14) : The locus C is $|2x^2 - (y - 1)^2| = 27$

The midpoint of RS is $(0, 1)$ and equation of perpendicular bisector which meet curve C at two distinct points $R'(a_1, b_1)$ and $S'(a_2, b_2)$ is $y=-\frac{x}{2}+1$

$\Rightarrow x + 2y - 2 = 0$

Solving it with curve C, we have

$|8(y - 1)^2 - (y - 1)^2| = 3\lambda^2$

$\Rightarrow\ |(y-1)^2|=\frac{3\lambda^2}{7}\ \therefore\ (y-1)=\pm\sqrt{\frac{3}{7}}\lambda$

$\Rightarrow\ y=1\pm\sqrt{\frac{3}{7}}\lambda\ \therefore\ b_1=1+\sqrt{\frac{3}{7}}\lambda$ and $b_2=1-\sqrt{\frac{3}{7}}\lambda$

∴ $D = (a_1 - a_2)^2 + (b_1 - b_2)^2$

$= 4(b_1 - b_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

$=5\left(2\sqrt{\frac{3}{7}}\lambda\right)^2=\frac{5\cdot 4\cdot 27}{7}=\frac{540}{7}=77.14$

[Topic : Co-ordinate Geometry]
[Rating : Medium]

11. (a, b, d) : We have,

$$EP=\begin{bmatrix}1&2&3\\2&3&4\\8&13&18\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}=\begin{bmatrix}1&3&2\\2&4&3\\8&18&13\end{bmatrix}$$

$$\text{Then } PEP=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&3&2\\2&4&3\\8&18&13\end{bmatrix}=\begin{bmatrix}1&3&2\\8&18&13\\2&4&3\end{bmatrix}=F$$

$$P^2=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$

So, (a) is true.

Clearly, $|E| = 0$, $|F| = 0$ and $|Q| \neq 0$ (Given)

Let $X=EQ+PFQ^{-1}$

Then $XQ = EQ^2 + PF = EQ^2 + PPEP$

$= EQ^2 + P^2EP = EQ^2 + EP = E(Q^2 + P)$

Now, $|XQ| = |E|\,|Q^2 + P| = 0$

$\Rightarrow\ |X|\,|Q| = 0$

$\Rightarrow\ |X| = 0$ (∵ $|Q| \neq 0$)

Also, $|EQ| + |PFQ^{-1}|$

$= |E|\,|Q| + |P|\,|F|\,|Q^{-1}| = 0 + 0 = 0$

So, (b) is true.

As $|(EF)^3| = 0$ and $|(EF)|^2 = 0$. So (c) is false.

Since, $\mathrm{tr}\,(A + B) = \mathrm{tr}\,A + \mathrm{tr}\,B$

$\mathrm{tr}\,(P^{-1}EP + F) = \mathrm{tr}\,(P^{-1}EP) + \mathrm{tr}F = \mathrm{tr}E + \mathrm{tr}F$

Also, $\mathrm{tr}\,(E + P^{-1}FP) = \mathrm{tr}E + \mathrm{tr}(P^{-1}FP) = \mathrm{tr}E + \mathrm{tr}F$

So, (d) is true.

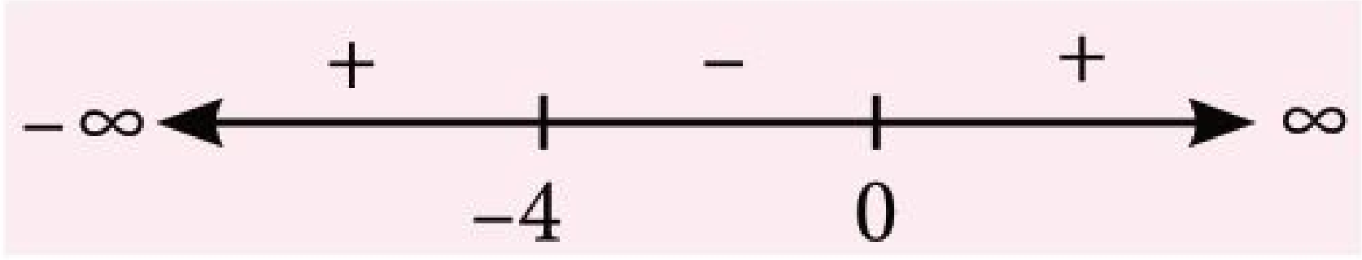
[Topic : Matrices]
[Rating : Difficult]

12. (a, b) : $f(x)=\frac{x^2-3x-6}{x^2+2x+4}=1-\frac{5(x+2)}{x^2+2x+4}$

$$\Rightarrow\ f'(x)=-5\left[\frac{(x^2+2x+4)\cdot 1-(x+2)(2x+2)}{(x^2+2x+4)^2}\right]$$

$$=\frac{5x(x+4)}{(x^2+2x+4)^2}$$

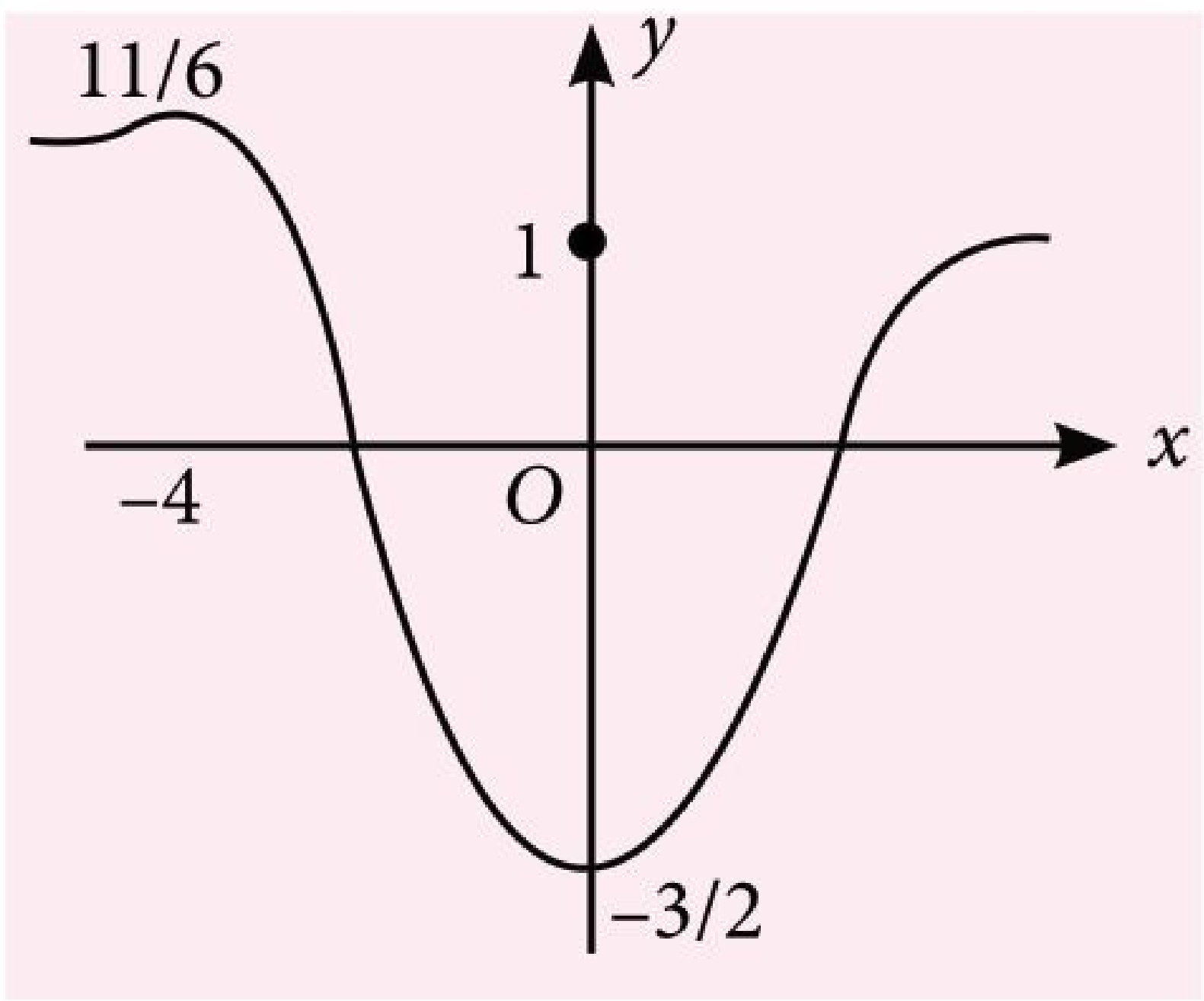
The sign scheme for $f'(x)$ is given as



Also, $f(0)=-\frac{3}{2};\ f(-4)=\frac{11}{6}$

$\lim\limits_{x\to\pm\infty} f(x)=1$

The graph of f is as shown :



The range of f is $\left[-\frac{3}{2},\frac{11}{6}\right]$

Clearly, f is not onto.

[Topic : Application of Derivatives]
[Rating : Easy]

13. (a, b, c) : (a) $P(E\cap F\cap G^c)=P(E\cap F)-P(E\cap F\cap G)$

$$\le P(E)-P(E\cap F\cap G)=\frac{1}{8}-\frac{1}{10}=\frac{1}{40}$$

(b) $P(E^c\cap F\cap G)=P(F\cap G)-(F\cap G\cap E)$

$$\le P(F)-P(E\cap F\cap G)=\frac{1}{6}-\frac{1}{10}=\frac{1}{15}$$

(c) $P(E\cup F\cup G)\le P(E)+P(F)+P(G)$

$$=\frac{1}{8}+\frac{1}{6}+\frac{1}{4}=\frac{3+4+6}{24}=\frac{13}{24}$$

(d) $P(E^c\cap F^c\cap G^c)=1-P(E\cup F\cup G)$

$$\le 1-\frac{13}{24}=\frac{11}{24}$$

[Topic : Probability]
[Rating : Medium]

14. (a, b, c) : Since, $G=(I-EF)^{-1}\Rightarrow FGE=F(I-EF)^{-1}E$
$=(E^{-1}(I-EF))F^{-1})^{-1}\Rightarrow FGE=(E^{-1}F^{-1}-I)^{-1}$
$\Rightarrow\ |FGE|\,|E^{-1}F^{-1}-I|=I$
$\Rightarrow\ |FGE|\,|E^{-1}F^{-1}-I|\,|FE|=|FE|\Rightarrow|FGE|\,|I-FE|=|FE|$
So, (a) is true.
We have, $G=(I-EF)^{-1}$
$\Rightarrow\ G^{-1}=I-EF$
Also, $G^{-1}G=I=GG^{-1}$
$\Rightarrow\ (I-EF)G=I=G(I-EF)\Rightarrow G-EFGI=G-GEF$
$\Rightarrow\ EFG=GEF$
So, (c) is true.
Also, $(I-FE)(I+FGE)=I+FGE-FE-FEFGE$
$=I+FGE-FE-F(G-I)E$
$=I+FGE-FE-FGE+FE=I$
So, (b) is true.

[Topic : Matrices]
[Rating : Difficult]

15. (a, b) : We have, $S_n(x)=\sum_{k=1}^{n}\tan^{-1}\left(\frac{x}{1+k(k+1)x^2}\right)$

$$=\sum_{k=1}^{n}\tan^{-1}\left(\frac{(kx+x)-(kx)}{1+(kx+x)(kx)}\right)$$

$$=\sum_{k=1}^{n}\tan^{-1}(k+1)x-\tan^{-1}kx$$

$$=\tan^{-1}(n+1)x-\tan^{-1}x=\tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)$$

$$\therefore\ S_{10}(x)=\tan^{-1}\left(\frac{10x}{1+11x^2}\right)=\frac{\pi}{2}-\cot^{-1}\left(\frac{10x}{1+11x^2}\right)$$

$$=\frac{\pi}{2}-\tan^{-1}\left(\frac{1+11x^2}{10x}\right)\forall\ x>0$$

So, (a) is true

$$\text{Now, }\lim_{n\to\infty}\cot\left(\tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)\right)$$

$$=\lim_{n\to\infty}\frac{1+(n+1)x^2}{nx}=\lim_{n\to\infty}\left[\frac{1}{nx}+x+\frac{x}{n}\right]=x$$

So, (b) is true.

$$\text{Now, }S_3(x)=\tan^{-1}4x-\tan^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow\ \tan^{-1}\frac{3x}{1+4x^2}=\frac{\pi}{4}\ \Rightarrow\ \frac{3x}{1+4x^2}=1$$

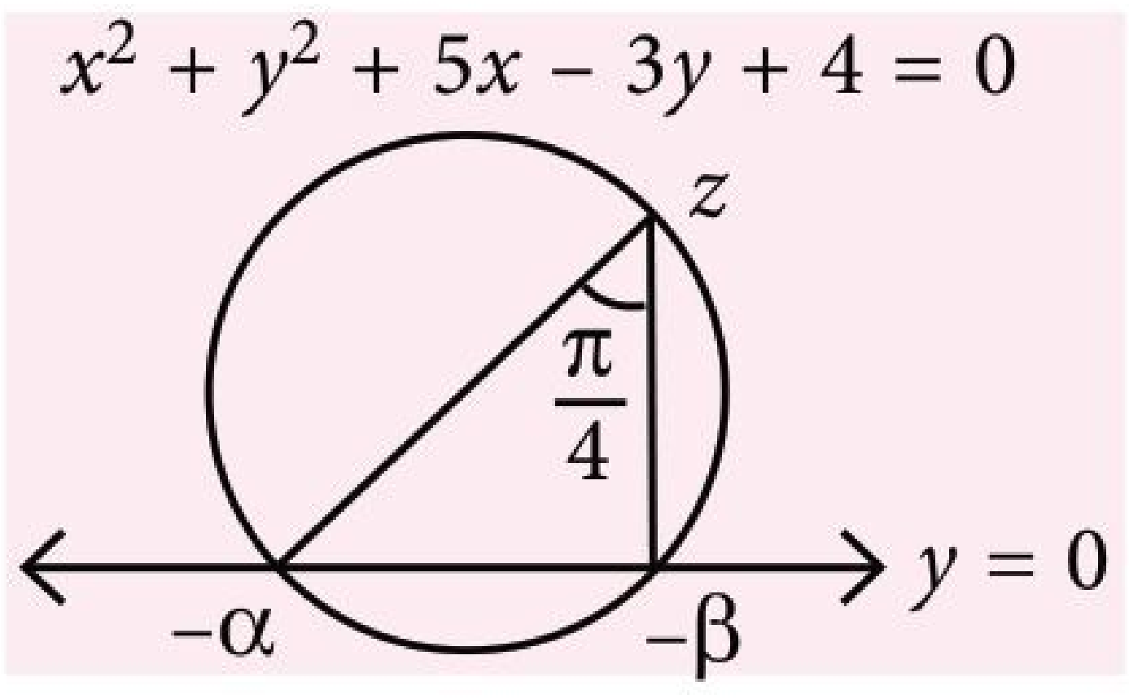
$\Rightarrow\ 4x^2-3x+1=0$
No real roots exist.

So, (c) is false.

$\tan(S_n(x))\to\frac{1}{x}$ when $n\to\infty$.

So, it doesn't have an upper bound for $x>0$.

[Topic : Inverse Trigonometric Functions]
[Rating : Easy]



16. (b, d) : As $y=0$ intersects the circle in points $(-\alpha,-\beta)$ when
$x^2+5x+4=0$
$\Rightarrow\ (x+1)(x+4)=0$
$\therefore\ x=-1,-4$
Thus, $\alpha=1$ and $\beta=4$

[Topic : Complex Numbers]
[Rating : Easy]

17. (4) : 1st solution :
For $x^2<1$ *i.e.*, $-1<x<1$
The equation $3x^2-|4x^2-4|+x-1=0$
$3x^2+4(x^2-1)+x-1=0$
$\Rightarrow\ 7x^2+x-5=0$

$$x=\frac{-1\pm\sqrt{141}}{14}$$

Both lie in $(-1, 1)$.
For $x^2\ge1$ *i.e.*, $x\ge1$ or $x\le-1$
The equation becomes
$3x^2-4(x^2-1)+x-1=0\Rightarrow -x^2+x+3=0$

$$\Rightarrow \quad x^2 - x - 3 = 0 \;\Rightarrow\; x = \frac{1 \pm \sqrt{13}}{2}$$

Both lie in $(-\infty, -1] \cup [1, \infty)$

So, 4 real solutions exist.

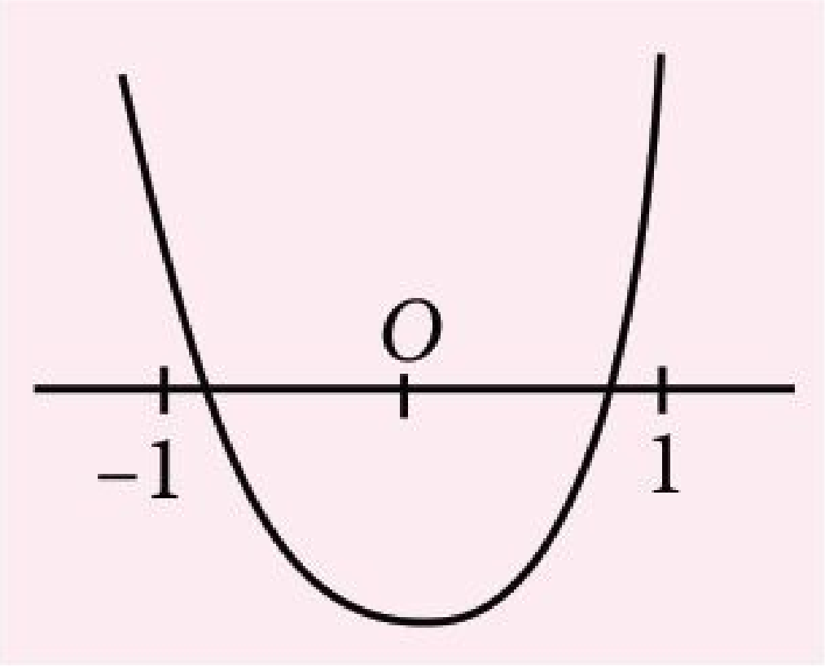
2$^{\text{nd}}$ solution : If $x \in [-1, 1]$, then equation is

$P(x) = 7x^2 + x - 5 = 0$

$P(1) = 3$

$P(-1) = 1$

$P(0) = -5$



And the graph of $P(x)$ is as shown.

So, 2 real roots.

If $x \in (-\infty, -1] \cup [1, \infty)$,

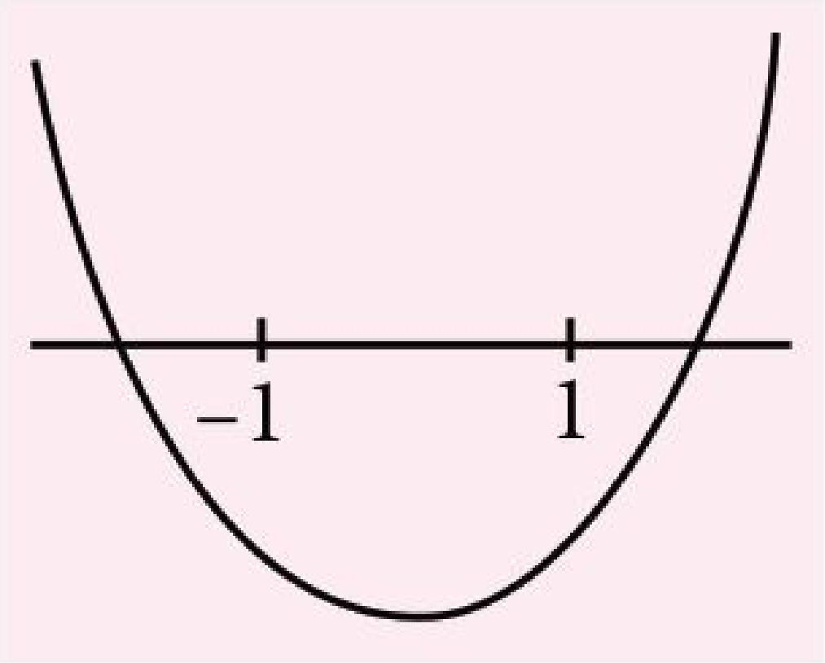
then equation is

$Q(x) = x^2 - x - 3 = 0$

$Q(-1) = -1$,

$Q(1) = -3$

$Q(3) = 3$



The graph of $Q(x)$ is as shown.

So, 2 real roots.

[Topic : Quadratic Equations]
[Rating : Easy]

18. (2) : Recall, $\cot A = \dfrac{\cos A}{\sin A} = \dfrac{\dfrac{b^2+c^2-a^2}{2bc}}{\sin A}$

$$= \frac{b^2+c^2-a^2}{2bc \sin A} = \frac{b^2+c^2-a^2}{4\Delta} \quad \left[\because\ \Delta = \frac{1}{2} bc \sin A\right]$$

Similarly, $\cot B = \dfrac{a^2+c^2-b^2}{4\Delta}$ and $\cot C = \dfrac{a^2+b^2-c^2}{4\Delta}$

$$\therefore \quad \frac{\cot A + \cot C}{\cot B} = \frac{\dfrac{b^2+c^2-a^2+a^2+b^2-c^2}{4\Delta}}{\dfrac{a^2+c^2-b^2}{4\Delta}}$$

$$= \frac{2b^2}{a^2+c^2-b^2} = \frac{2 \times 16}{9+23-16} = \frac{32}{16} = 2$$

[Topic : Solution of Triangles]
[Rating : Easy]

19. (7) : We have, $|\vec{u}| = 1, |\vec{v}| = 1, \vec{w} \cdot \vec{w} = 4 \Rightarrow |\vec{w}| = 2$

$\vec{u} \cdot \vec{w} = 1,\ \vec{v} \cdot \vec{w} = 1,\ \vec{u} \cdot \vec{v} = x$ (say)

$$\text{Now, } [\vec{u}\ \vec{v}\ \vec{w}]^2 = \begin{vmatrix} \vec{u}\cdot\vec{u} & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{v}\cdot\vec{u} & \vec{v}\cdot\vec{v} & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & \vec{w}\cdot\vec{w} \end{vmatrix}$$

$$\Rightarrow \; 2 = \begin{vmatrix} 1 & x & 1 \\ x & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix}$$

$\Rightarrow \quad 2 = 1(4 - 1) - x(4x - 1) + 1(x - 1)$

$\Rightarrow \quad 2 = 3 - 4x^2 + x + x - 1$

$\Rightarrow \quad 4x^2 - 2x = 0 \;\Rightarrow\; x(2x - 1) = 0$

$\Rightarrow \quad x = 1/2 \qquad [\because\ \vec{u} \cdot \vec{v} \neq 0 \text{ so } x \neq 0]$

$$\therefore\ |3\vec{u} + 5\vec{v}| = \sqrt{9|\vec{u}|^2 + 25|\vec{v}|^2 + 2 \cdot 3 \cdot 5\, \vec{u} \cdot \vec{v}}$$

$$= \sqrt{9 + 25 + 30\left(\frac{1}{2}\right)} = \sqrt{49} = 7$$

[Topic : Vector Algebra]
[Rating : Easy]

PAPER-2

1. (a, b, d) : $n_1 = 10 \cdot 10 \cdot 10 = 1000$, as each of i, j and k have 10 choices.

For S_2, we do casework on j.

Given, $i < j + 2$ *i.e.*, $i \le j + 1$

$j = 1 \quad i = 1, 2 \qquad \rightarrow$ 2 choices

$j = 2 \quad i = 1, 2, 3 \qquad \rightarrow$ 3 choices

....

$j = 8 \quad i = 1, 2, 3, \ldots, 9 \qquad \rightarrow$ 9 choices

Hence, $n_2 = 2 + 3 + .. + 9 = (1 + 2 + \ldots + 9) - 1$

$$= \frac{9 \times 10}{2} - 1 = 45 - 1 = 44$$

$$n_3 = {}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{24} = 210$$

$n_4 = {}^{10}C_4 \cdot 4! = 210 \cdot 4!$

Hence, $\dfrac{n_4}{12} = 420$

[Topic : Permutations and Combinations]
[Rating : Easy]

2. (a, b) : As $\cos P = \dfrac{q^2+r^2-p^2}{2qr} = \dfrac{q^2+r^2}{2qr} - \dfrac{p^2}{2qr}$

$$\ge 1 - \frac{p^2}{2qr}$$

Hence, (a) is true.

Note that $\dfrac{q^2+r^2}{2qr} \ge 1$ [Using A.M. ≥ G.M.]

Now, $\cos R \ge \dfrac{q-r}{p+q} \cos P + \dfrac{p-r}{p+q} \cos Q$

$\Rightarrow \quad (p + q)\cos R \ge (q\cos P + p\cos Q) - r(\cos P + \cos Q)$

$\Rightarrow \quad (p\cos R + r\cos P) + (q\cos R + r\cos Q) \ge q\cos P + p\cos Q$

$\Rightarrow \quad q + p \ge r,$

which is the triangle inequality. Hence, (b) is true.

Again, $\dfrac{q+r}{p}=\dfrac{\sin Q+\sin R}{\sin P}\geq\dfrac{2\sqrt{\sin Q\sin R}}{\sin P}$

So, (c) is false.

Finally, $\cos Q>\dfrac{p}{r}\Rightarrow \sin R\cos Q>\sin P$

$\Rightarrow\quad \sin R\cos Q>\sin(Q+R)$

$\Rightarrow\quad 0>\cos R\sin Q$, which in general is not true.

Hence, (d) is false.

[Topic : Solution of Triangles]
[Rating : Medium]

3. (a, b, c) : Let $g(x)=f(x)-3\cos 3x$

$$\therefore\ \int_0^{\pi/3} g(t)\,dt=\int_0^{\pi/3}(f(t)-3\cos 3t)\,dt$$

$$=\int_0^{\pi/3} f(t)dt-3\int_0^{\pi/3}\cos 3t\,dt=0-\left[3\frac{\sin 3t}{3}\right]_0^{\pi/3}=0$$

Hence, $g(x)=0$ has at least one solution in $(0,\pi/3)$.

Again, let $h(x)=f(x)-3\sin 3x+\dfrac{6}{\pi}$

$$\therefore\ \int_0^{\pi/3} h(t)dt=\int_0^{\pi/3}\left(f(t)-3\sin 3t+\frac{6}{\pi}\right)dt$$

$$=\int_0^{\pi/3} f(t)dt-3\int_0^{\pi/3}\sin 3t\,dt+\frac{6}{\pi}\int_0^{\pi/3}dt$$

$$=0+\left[3\frac{\cos 3t}{3}\right]_0^{\pi/3}+\frac{6}{\pi}\cdot\frac{\pi}{3}=-1-1+2=0$$

Hence, $h(x)=0$ has at least one solution in $(0,\pi/3)$.

Now, $$\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{1-e^{x^2}}=\lim_{x\to 0}\frac{x^2}{1-e^{x^2}}\cdot\frac{\int_0^x f(t)dt}{x}$$

$$=\left(\lim_{x\to 0}\frac{1}{\frac{1-e^{x^2}}{x^2}}\right)\left(\lim_{x\to 0}\frac{\int_0^x f(t)dt}{x}\right)$$

$$=-1\cdot\lim_{x\to 0}\frac{f(x)}{1}=-f(0)=-1$$

Finally, $$\lim_{x\to 0}\frac{\sin x\int_0^x f(t)dt}{x^2}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\int_0^x f(t)dt}{x}$$

$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{\int_0^x f(t)dt}{x}\right)=1\cdot\lim_{x\to 0}\frac{f(x)}{1}=f(0)=1$$

[Topic : Limit, Continuity and Differentiability]
[Rating : Medium]

4. (a,c) : The given differential equation is

$$\frac{dy}{dx}+\alpha y=xe^{\beta x}$$

Integrating factor $=e^{\alpha\int dx}=e^{\alpha x}$

$\therefore$ The solution is given by

$ye^{\alpha x}=\int xe^{(\alpha+\beta)x}dx+\lambda$, where λ is a constant.

If $\alpha+\beta=0$, then $ye^{\alpha x}=\dfrac{x^2}{2}+\lambda$

Given, $y(1)=1\Rightarrow e^{\alpha}=\dfrac{1}{2}+\lambda\Rightarrow\lambda=e^{\alpha}-\dfrac{1}{2}$

$\therefore$ From (i), we have, $ye^{\alpha x}=\dfrac{x^2}{2}+e^{\alpha}-\dfrac{1}{2}$

$$\Rightarrow\quad y=\left(\frac{x^2}{2}+e^{\alpha}-\frac{1}{2}\right)e^{-\alpha x}$$

For $\alpha=1$, we get option (a).

If $\alpha+\beta\neq 0$, then

$$ye^{\alpha x}=\int xe^{(\alpha+\beta)x}dx+\lambda$$

$$\Rightarrow\quad ye^{\alpha x}=\frac{xe^{(\alpha+\beta)x}}{\alpha+\beta}-\frac{1}{(\alpha+\beta)^2}e^{(\alpha+\beta)x}+\lambda.$$

Also, $y(1)=1$ gives

$$e^{\alpha}=\frac{e^{\alpha+\beta}}{\alpha+\beta}-\frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}+\lambda$$

$$\Rightarrow\quad \lambda=e^{\alpha}-\frac{e^{\alpha+\beta}}{\alpha+\beta}+\frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}$$

$\therefore$ y is given by

$$y=\frac{xe^{\beta x}}{\alpha+\beta}-\frac{e^{\beta x}}{(\alpha+\beta)^2}+\left(e^{\alpha}-\frac{e^{\alpha+\beta}}{\alpha+\beta}+\frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}\right)e^{-\alpha x}$$

For $\alpha=\beta=1$, we get

$$y=(2x-1)\frac{e^x}{4}+\left(e-\frac{e^2}{2}+\frac{e^2}{4}\right)e^{-x}$$

$=\left(x-\dfrac{1}{2}\right)\dfrac{e^x}{2}+\left(e-\dfrac{e^2}{4}\right)e^{-x}$, which is option (c).

[Topic : Differential Equations]
[Rating : Medium]

5. (a, b, c) : We have, $\overrightarrow{OB}\times\overrightarrow{OC}=\overrightarrow{OB}\times\left(\dfrac{\overrightarrow{OB}-\lambda\overrightarrow{OA}}{2}\right)$

$$=-\frac{\lambda}{2}(\overrightarrow{OB}\times\overrightarrow{OA})=-\frac{\lambda}{2}(6\hat{i}-3\hat{j}-6\hat{k})$$

$$=\frac{3\lambda}{2}(-2\hat{i}+\hat{j}+2\hat{k})$$

Given, $|\overrightarrow{OB}\times\overrightarrow{OC}|=\frac{9}{2}$

$$\Rightarrow\ \frac{9}{2}=\frac{3}{2}\lambda\sqrt{(-2)^2+1^2+2^2}\quad(\because\ \lambda>0)$$

$$\Rightarrow\ \frac{9}{2}=\frac{3}{2}\lambda\cdot 3\ \Rightarrow\ \lambda=1$$

Now, $\overrightarrow{OC}=\frac{1}{2}(\overrightarrow{OB}-\overrightarrow{OA})=\frac{-\hat{i}-4\hat{j}+\hat{k}}{2}$

Now, projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$

$$=\overrightarrow{OC}\cdot\frac{\overrightarrow{OA}}{|\overrightarrow{OA}|}=\frac{-2-8+1}{2\cdot 3}=-\frac{3}{2}$$

$$\text{Area of } \Delta OAB=\frac{1}{2}\cdot|\overrightarrow{OA}\times\overrightarrow{OB}|=\frac{1}{2}\times 9=\frac{9}{2}$$

$$\text{Area of } \Delta ABC=\frac{1}{2}\cdot|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}|2\overrightarrow{OC}\times(\overrightarrow{OC}-\overrightarrow{OA})|$$

$$=\frac{1}{2}\cdot 2|\overrightarrow{OC}\times\overrightarrow{OA}|=|\overrightarrow{OC}\times\overrightarrow{OA}|=\frac{9}{2}$$

The diagonals are $\overrightarrow{OA}+\overrightarrow{OC}$ and $\overrightarrow{OA}-\overrightarrow{OC}$

i.e., $\frac{3}{2}(\hat{i}+\hat{k})$ and $\frac{1}{2}(5\hat{i}+8\hat{j}+\hat{k})$

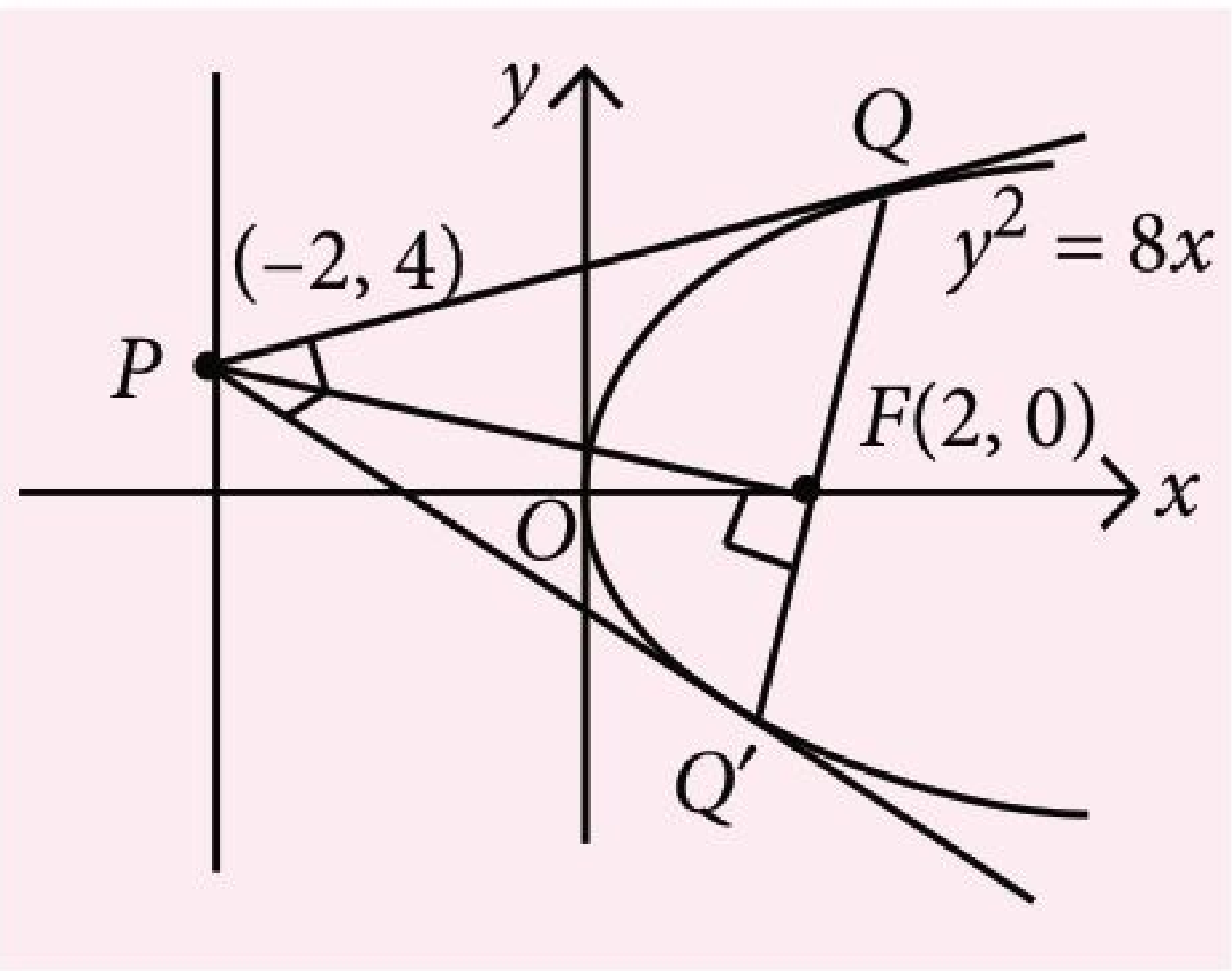
The angle between them is given by

$$\cos\theta=\frac{1\cdot 5+0\cdot 8+1\cdot 1}{\sqrt{2}\sqrt{90}}=\frac{6}{\sqrt{180}}=\frac{6}{6\sqrt{5}}=\frac{1}{\sqrt{5}}$$

[Topic : Vector Algebra]
[Rating : Easy]

6. (a, b, d) : The parabola $y^2 = 8x$ has its directrix at $x = -2$.

The point $P(-2, 4)$ lies on the directrix. So each of the tangents, drawn from P to the parabola, *i.e.*, Q and Q' will be the ends of a focal chord. Also, $\angle QPQ' = 90°$.



Also, it is known that the foot of the perpendicular from P on QQ' is focus F.

Now distance, $PF=\sqrt{4^2+4^2}=4\sqrt{2}$

[Topic : Parabola]
[Rating : Medium]

(7-8) : Let $P(4 - k^2, k)$ be the parametric co-ordinates of parabola.

Equation of normal to parabola at $P(4 - k^2, k)$ is

$$y - k = 2k(x - (4 - k^2))$$

As it passes through centre $(a, 0)$.

$\therefore\quad (0 - k) = 2k(a - (4 - k^2))$

$$\Rightarrow\ a=\frac{7}{2}-k^2>0\ \Rightarrow\ k^2<\frac{7}{2}$$

The equation of circle in the form $S + \lambda L = 0$ gives

$$(x-(4-k^2))^2+(y-k)^2+\lambda\left(\frac{x}{2k}+y-k-\frac{2}{k}+\frac{k}{2}\right)=0$$

As centre lies on x-axis, we get

$$k-\frac{\lambda}{2}=0.\ \therefore\ \ \lambda=2k$$

Now (0, 0) should lie on or outside circle, therefore

$S_1 \geq 0 \Rightarrow (4 - k^2)^2 - 4 \geq 0 \Rightarrow (k^2 - 2)(k^2 - 6) \geq 0$

$\Rightarrow k^2 \leq 2$ or $k^2 \geq 6$ [not possible]

$$\therefore \text{Radius of circle } C=\sqrt{g^2-c}=\sqrt{\left(\frac{7}{2}-k^2\right)^2-k^4+8k^2-12}$$

$$=\sqrt{k^2+\frac{1}{4}}$$

when $k^2 = 2$ then $r_{\max}=\sqrt{2+\frac{1}{4}}=\frac{3}{2}=1.5$

And then x-coordinate of point of intersection is

$$\alpha = 4 - k^2 = 4 - 2 = 2$$

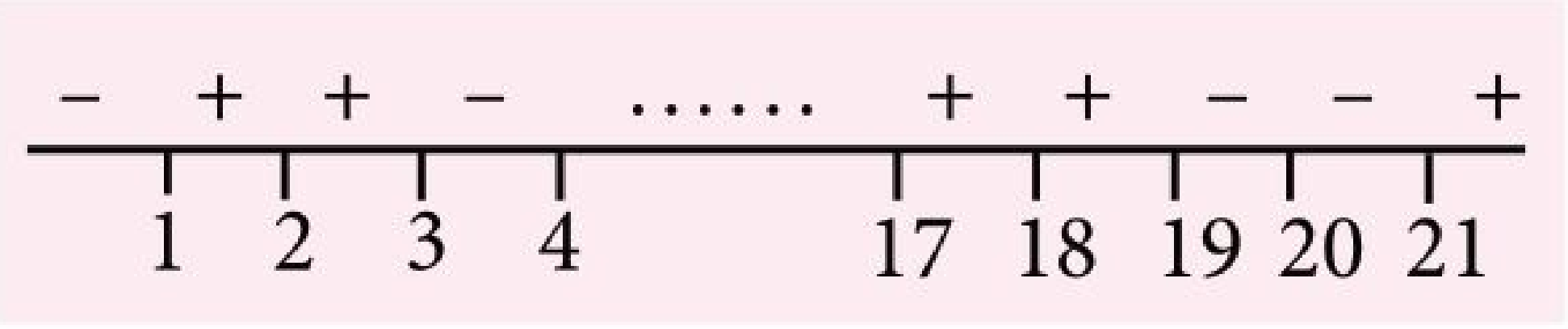
7. (1.5) **8. (2)**

[Topic : Circle and Parabola]
[Rating : Challenging]

(9-10) : $f_1(x)=\int_0^x\prod_{j=1}^{21}(t-j)^j\,dt,\ x>0$

$$\Rightarrow\ f_1'(x)=\prod_{j=1}^{21}(x-j)^j=(x-1)(x-2)^2(x-3)^3....(x-21)^{21}$$

The sign scheme for $f_1'(x)$ is given below



The function has minima at x = 1, 5, 9, 13, 17, 21.

Hence, $m_1 = 6$

The function has maxima at x = 3, 7, 11, 15, 19.

Hence, $n_1 = 5$

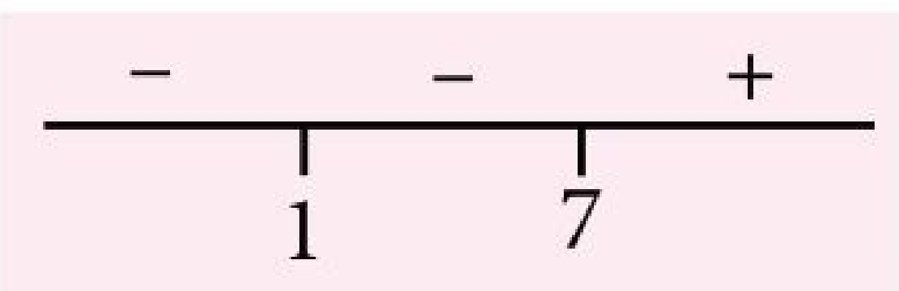
Now, $2m_1 + 3n_1 + m_1n_1 = 12 + 15 + 30 = 57$

Next, $f_2(x) = 98(x - 1)^{50} - 600(x - 1)^{49} + 2450,\ x > 0$

$\Rightarrow f_2'(x) = 98\cdot 50(x - 1)^{49} - 600\cdot 49(x - 1)^{48}$

$= 980\cdot 5(x - 1)^{48}(x - 1 - 6) = k(x - 1)^{48}(x - 7)$

The sign scheme for $f_2'(x)$ is given as :



It has a minima at $x = 7$.

So, $m_2 = 1$

Also, there is no maxima.

So, $n_2 = 0$.

Hence, $6m_2 + 4n_2 + 8m_2n_2 = 6 + 0 + 0 = 6$

9. (57) **10. (6)**

[Topic : Application of Derivatives]
[Rating : Medium]

(11-12) : $S_1 = \int_{\pi/8}^{3\pi/8} \sin^2 x dx = \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{8}+\frac{3\pi}{8}-x\right)dx$

$= \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{2}-x\right)dx = \int_{\pi/8}^{3\pi/8} \cos^2 x dx$

We have, $2S_1 = \int_{\pi/8}^{3\pi/8} 1\, dx = \frac{3\pi}{8}-\frac{\pi}{8}=\frac{\pi}{4}$

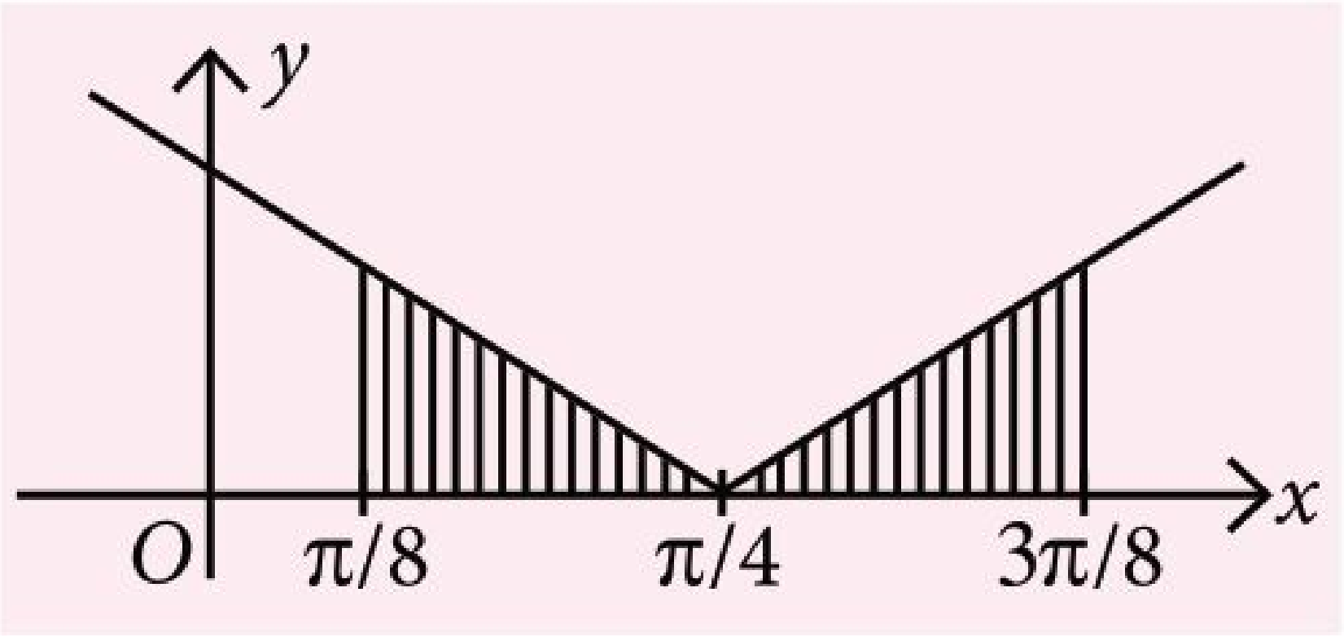
Thus, $S_1 = \frac{\pi}{8}$. Hence, $\frac{16S_1}{\pi}=2$

Now, $S_2 = \int_{\pi/8}^{3\pi/8} \sin^2 x\,|4x-\pi|\,dx \;= 4\int_{\pi/8}^{3\pi/8} \sin^2 x\left|\frac{\pi}{4}-x\right|dx$

$= 4\int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{2}-x\right)\left|\frac{\pi}{4}-\left(\frac{\pi}{2}-x\right)\right|dx$

$\Rightarrow S_2 = 4\int_{\pi/8}^{3\pi/8} \cos^2 x\left|x-\frac{\pi}{4}\right|dx \;\therefore\; 2S_2 = 4\int_{\pi/8}^{3\pi/8}\left|x-\frac{\pi}{4}\right|dx$

The graph of $\left|x-\frac{\pi}{4}\right|$ is given below :



$\therefore \quad 2S_2 = 2\cdot\frac{1}{2}\cdot\frac{\pi}{8}\cdot\frac{\pi}{2}=\frac{\pi^2}{16} \Rightarrow 2S_2 = \frac{\pi^2}{16}$

Hence, $\frac{48S_2}{\pi^2}=\frac{24}{16}=\frac{3}{2}=1.5$

11. (2) **12. (1.5)**

[Topic : Definite Integration]
[Rating : Medium]

13. (d) : We have, $S_n = 1+\frac{1}{2}+\frac{1}{2^2}+......+\frac{1}{2^{n-1}}$

$= \frac{1-\frac{1}{2^n}}{1-\frac{1}{2}} = 2\left(1-\frac{1}{2^n}\right)=2-\frac{1}{2^{n-1}}$

The radius of C_n is $\frac{1}{2^{n-1}}$ and its center is $\left(2-\frac{1}{2^{n-2}},0\right)$.

For the case, where $r=\frac{1025}{513}(<2)$

We know that C_n will lie inside M if

$$2-\frac{1}{2^{n-2}}+\frac{1}{2^{n-1}}<\frac{1025}{513}$$

The bound for n becomes $n \le 10$.
So $k = 10$.
Also, the maximum number of circles such that no two circles intersect = 5. ∴ $l = 5$.
So, $3k + 2l = 30 + 10 = 40$

14. (b) : Centre of D_n is (S_{n-1}, S_{n-1})

$$\sqrt{2}S_{n-1} < \sqrt{2}\cdot\frac{2^{199}-1}{2^{198}}$$

$$\Rightarrow \frac{\sqrt{2}}{2^{n-2}} > \frac{\sqrt{2}}{2^{198}}+\frac{1}{2^{n-1}}$$

This gives $n_{\max} = 199$

[Topic : Circle]
[Rating : Challenging]

15. (c) : $f(x)=\int_{-x}^{x}(|t|-t^2)e^{-t^2}dt$

$$=2\int_0^x (t-t^2)e^{-t^2}dt \qquad ...(i)$$

Also, $g(x)=\int_0^{x^2}\sqrt{t}\,e^{-t}dt$

Let $t = u^2$, then

$$g(x)=2\int_0^x u^2e^{-u^2}du=2\int_0^x t^2e^{-t^2}dt \qquad ...(ii)$$

Adding (i) and (ii), we get

$$f(x)+g(x)=2\int_0^x te^{-t^2}dt=-\int_0^x e^{-t^2}d(-t^2)$$

$$=\left[-e^{-t^2}\right]_0^x=1-e^{-x^2}$$

So, $(f\sqrt{\ln 3}+g\sqrt{\ln 3})=1-e^{-\ln 3}=1-\frac{1}{3}=\frac{2}{3}$

Also, $f'(x) = 2(x - x^2)e^{-x^2} = 2x(1 - x)e^{-x^2}$
Thus, $f'(x) > 0 \;\forall\; x \in (0, 1)$
$\psi_1(x) > e^{-x} + x$
So, $\psi_1'(x) = -e^{-x} + 1$
$\psi'(x) > 0$, if $x > 1$; $\psi_1'(x)\, x < x$, $\psi_1'(\infty) = 1$
But $\psi_1'(x) < 1 \;\forall\; x \in R$
So, $\psi_1(x)$ can't be equal to $1 + \alpha x$, for some $\alpha \in (1, x)$
Applying Mean Value Theorems to $\psi_2(x)$ in $(0, x)$, we get

$$\frac{\psi_2(x)-\psi_2(0)}{x-0}=\psi_2'(\beta) \text{ for } \beta\in(0,x)$$

$$\Rightarrow \frac{\psi_2(x)-0}{x}=2\beta-2+2e^{-\beta}$$

So, $\psi_2(x) = 2x(e^{-\beta} + \beta - 1) = 2x(\psi_1(\beta) - 1)$, which is true.

16. (d) : $\psi_1(x) = e^{-x} + x,\ x \geq 0.$

$\Rightarrow \quad \psi_1'(x) = 1 - e^{-x} > 0$

Now, $\psi_1(x) \geq 1\ \forall\ x \in R.$

Also, $\psi_2'(x) = 2x - 2 + 2e^{-x}$

$\Rightarrow \quad 2\psi_1(x) - 2 \geq 0\ \forall\ x \geq 0$

$\Rightarrow \quad \psi_2(x)$ is increasing. So, $\psi_2(x) \geq 0$

Now, $f(x) = 2\int_0^x (t - t^2)e^{-t^2}\,dt$

$$= \left[-e^{-t^2}\right]_0^x - 2\int_0^x t^2 e^{-t^2}\,dt$$

$$= 1 - e^{-x^2} - 2\int_0^x t^2\left(1 - t^2 + \frac{t^4}{2!} - .....\right)dt$$

$$= 1 - e^{-x^2} - \frac{2x^3}{3} + \frac{2x^5}{5} - \frac{2x^7}{7\cdot 2} + \frac{2x^9}{9\cdot 6} - ...$$

Thus, $f(x) - 1 + e^{-x^2} + \frac{2x^3}{3} - \frac{2x^5}{5} \leq 0\ \forall\ x \in \left(0, \frac{1}{2}\right)$

Finally, $g(x) = \int_0^{x^2} \sqrt{t}\, e^{-t}\,dt = \int_0^x 2u^2 e^{-u^2}\,du$

$$= \int_0^x 2u^2\left(1 - u^2 + \frac{u^4}{2!} - ...\right)du$$

$$= \frac{2x^3}{3} - \frac{2x^5}{5} + \frac{2x^7}{2\cdot 7} - \frac{2x^9}{6\cdot 9} + ...$$

Thus, $g(x) - \frac{2x^3}{3} + \frac{2x^5}{5} - \frac{x^7}{7} \leq 0$ for all $x \in \left(0, \frac{1}{2}\right)$

[Topic : Definite Integration]
[Rating : Challenging]

17. (214) : Let $|M(k)|$ = Number of multiples of k in the given set. Then

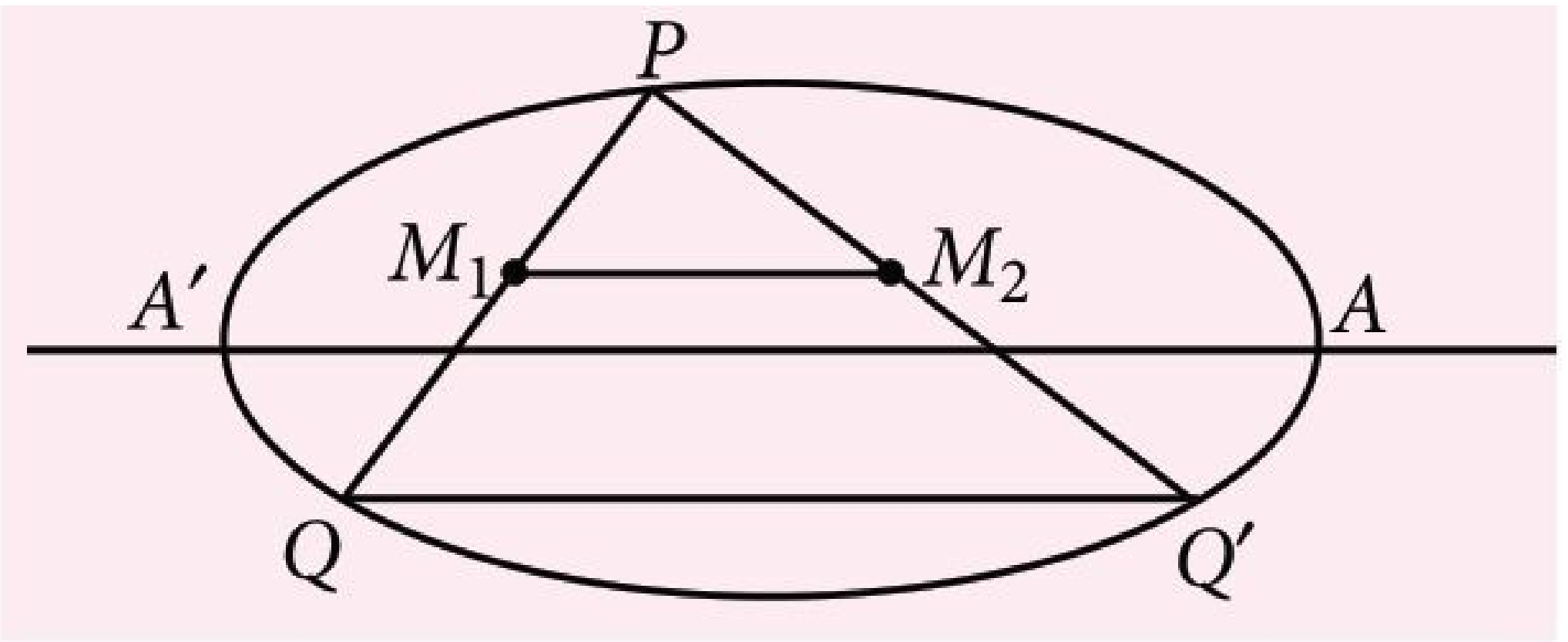
$$|M(3)| = \left[\frac{2000}{3}\right] = 666\ ;\ |M(7)| = \left[\frac{2000}{7}\right] = 285$$

$$|M(21)| = \left[\frac{2000}{21}\right] = 95$$

The probability, $p = \frac{666 + 285 - 95}{2000} = \frac{856}{2000} = \frac{214}{500}$

Thus, $500p = 214$

18. (4) :



Let $M_1 \rightarrow$ midpoint of PQ

$M_2 \rightarrow$ midpoint of PQ'

We have, $M_1M_2 = \frac{1}{2}QQ'.$ To get the maximum value of M_1M_2, we can choose Q, Q' to be the end of the major axis.

Hence, $M_1M_{2\,\max} = \frac{1}{2}AA' = \frac{1}{2}\cdot 8 = 4$

[Topic : Ellipse]
[Rating : Easy]

19. (182) : For, $f(x) = \frac{10x}{x+1} = 10\left(1 - \frac{1}{x+1}\right)$

$f'(x) = \frac{10}{(x+1)^2} > 0\ \forall\ x \in [0, 10]$

Hence, $f(x)$ is an increasing function

$$\left[\sqrt{\frac{10x}{x+1}}\right] = 0, \text{ if } 0 \leq \sqrt{\frac{10x}{x+1}} < 1 \quad i.e.,\ 0 \leq x < \frac{1}{9}$$

$$\left[\sqrt{\frac{10x}{x+1}}\right] = 1, \text{ if } 1 \leq \sqrt{\frac{10x}{x+1}} < 2 \quad i.e.,\ \frac{1}{9} \leq x < \frac{2}{3}$$

$$\left[\sqrt{\frac{10x}{x+1}}\right] = 2, \text{ if } 2 \leq \sqrt{\frac{10x}{x+1}} < 3 \quad i.e.,\ \frac{2}{3} \leq x < 9$$

Now, $I = \int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx = \int_0^{1/9} 0dx + \int_{1/9}^{2/3} 1dx + \int_{2/3}^{9} 2dx + \int_9^{10} 3dx$

$$= \left(\frac{2}{3} - \frac{1}{9}\right) + 2\left(9 - \frac{2}{3}\right) + 3(10 - 9)$$

$$= \frac{2}{3} - \frac{1}{9} + 18 - \frac{4}{3} + 30 - 27 = -\frac{2}{3} - \frac{1}{9} - 9 + 30$$

$$= \frac{-6 - 1 - 81 + 270}{9} = \frac{182}{9}$$

Thus, $9I = 182$

[Topic : Definite Integration]
[Rating : Medium

⬥⬥

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SOLVED PAPER

Held on
1st September, Evening
Shift

JEE MAIN 2021

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to (The inverse trigonometric functions takes the principal values)

(a) $3\pi + 1$ (b) $3\pi - 11$
(c) $4\pi - 11$ (d) $4\pi - 9$

2. Let $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \ldots + (n-1)\cdot 1$, $n \geq 4$. The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to

(a) $\frac{e-2}{6}$ (b) $\frac{e-1}{3}$ (c) $\frac{e}{6}$ (d) $\frac{e}{3}$

3. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is

(a) 6 (b) 8 (c) 4 (d) 2

4. The range of the function

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$ is

(a) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (b) $[0, 2]$
(c) $(0, \sqrt{5})$ (d) $[-2, 2]$

5. Let $f : R \to R$ be a continuous function. Then

$$\lim_{x \to \pi/4} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)\,dx}{x^2 - \frac{\pi^2}{16}}$$ is equal to

(a) $f(2)$ (b) $2f(\sqrt{2})$ (c) $2f(2)$ (d) $4f(2)$

6. If $y = y(x)$ is the solution curve of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$, and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to

(a) $3 + e$ (b) $3 - e$
(c) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (d) $3 + \frac{1}{\sqrt{e}}$

7. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to

(a) $\frac{25}{2}$ (b) $\frac{75}{8}$ (c) $\frac{125}{16}$ (d) $\frac{15}{2}$

8. Consider the system of linear equations

$-x + y + 2z = 0$
$3x - ay + 5z = 1$
$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(a) $n(S_1) = 2$, $n(S_2) = 0$ (b) $n(S_1) = 2$, $n(S_2) = 2$
(c) $n(S_1) = 0$, $n(S_2) = 2$ (d) $n(S_1) = 1$, $n(S_2) = 0$

9. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point $(2, -1, 6)$ is

(a) $2\sqrt{5}$ (b) $2\sqrt{6}$ (c) $\sqrt{26}$ (d) $4\sqrt{2}$

10. The area, enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ and the lines $x = 0$, $x = \frac{\pi}{2}$, is

(a) $2\sqrt{2}(\sqrt{2}+1)$ (b) $2\sqrt{2}(\sqrt{2}-1)$
(c) $4(\sqrt{2}-1)$ (d) $2(\sqrt{2}+1)$

11. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1$, $x \in [0, \pi]$ and

S is the sum of all these solutions, then the ordered pair (n, S) is

(a) $(2, 8\pi/9)$ (b) $(3, 13\pi/9)$
(c) $(2, 2\pi/3)$ (d) $(3, 5\pi/3)$

12. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S_1) there exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S_2) there exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

Then

(a) (S_1) is true and (S_2) is false
(b) both (S_1) and (S_2) are false
(c) both (S_1) and (S_2) are true
(d) (S_1) is false and (S_2) is true

13. Let $J_{n,m} = \int\limits_0^{1/2} \frac{x^n}{x^m - 1}\,dx, \forall\, n > m$ and $n, m \in N$.

Consider a matrix $A = [a_{ij}]_{3\times 3}$, where

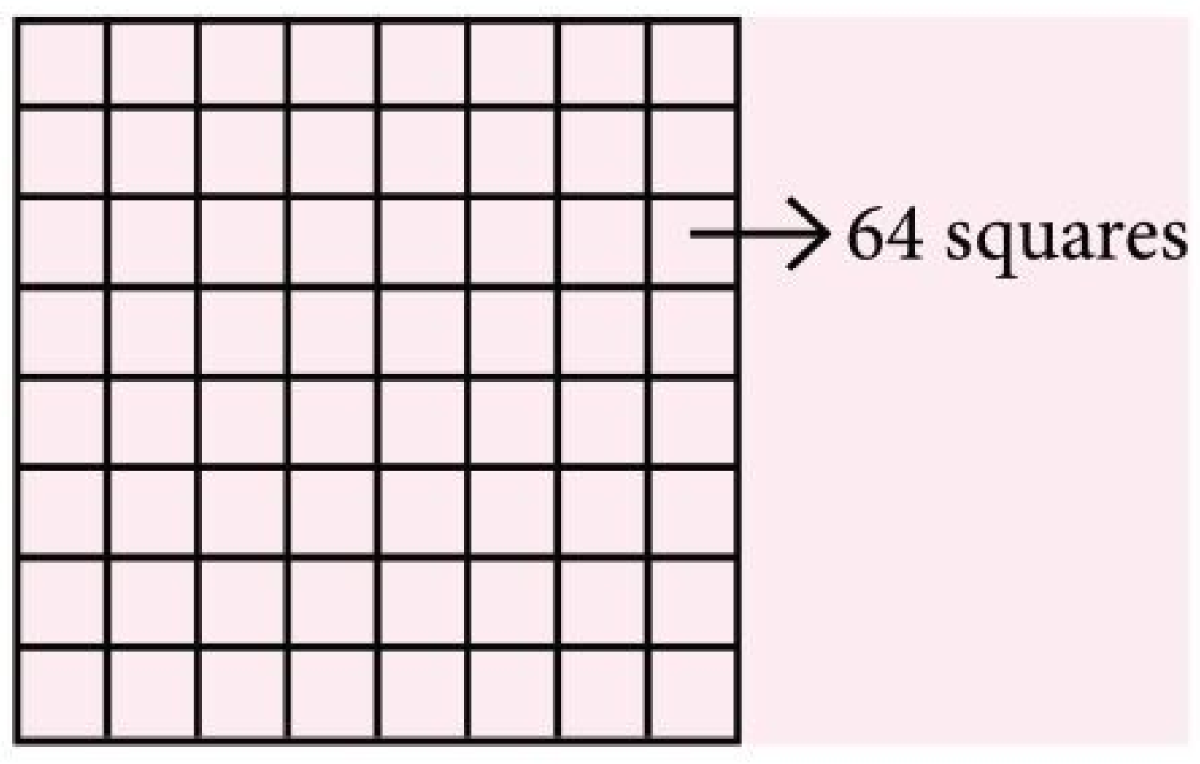
$$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}.$$ Then $|\text{adj } A^{-1}|$ is

(a) $(15)^2 \times 2^{34}$ (b) $(15)^2 \times 2^{42}$
(c) $(105)^2 \times 2^{36}$ (d) $(105)^2 \times 2^{38}$

14. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P ?

(a) $(0, 2, -4)$ (b) $\left(-2, 0, -\frac{1}{2}\right)$
(c) $(4, 0, -2)$ (d) $\left(3, 1, -\frac{1}{2}\right)$

15. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is



(a) $\frac{1}{9}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{18}$

16. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(a) $\sim p \rightarrow \sim q$ (b) $\sim (q \rightarrow p)$
(c) $\sim (p \rightarrow q)$ (d) $\sim (p \rightarrow \sim q)$

17. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to

(a) $\frac{5}{2\sqrt{3}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) 2

18. The function $f(x)$, that satisfies the condition

$f(x) = x + \int\limits_0^{\pi/2} \sin x \cdot \cos y\, f(y)\,dy$, is

(a) $x + \frac{\pi}{2}\sin x$ (b) $x + (\pi + 2)\sin x$
(c) $x + \frac{2}{3}(\pi - 2)\sin x$ (d) $x + (\pi - 2)\sin x$

19. Let $P_1, P_2 ..., P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \ne 15$, is

(a) 455 (b) 419 (c) 12 (d) 443

20. Let $a_1, a_2, ..., a_{21}$ be an AP such that $\sum\limits_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this AP is 189, then $a_6 a_{16}$ is equal to

(a) 57 (b) 48 (c) 36 (d) 72

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is ________ .

22. Let $f(x)$ be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10\, f(10)$ is equal to ________ .

23. Let $[t]$ denote the greatest integer $\le t$. The number of points where the function

$f(x) = [x]\,|x^2 - 1| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$ is

not continuous is ________ .

24. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ________ .

25. A man starts walking from the point $P(-3, 4)$, touches the x-axis at R, and then turns to reach at the point $Q(0, 2)$. The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50((PR)^2 + (RQ)^2)$ is equal to ________ .

26. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, $x \in R$. Then the natural number n for which $\lim\limits_{x\to 1} \dfrac{x^n f(1) - f(x)}{x-1} = 44$ is ________ .

27. Let X be random variable with distribution.

x	-2	-1	3	4	6
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\,\sigma^2$ is equal to ________ .

28. If for the complex numbers z satisfying $|z - 2 - 2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to ________ .

29. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is ________ .

30. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to ________ .

SOLUTIONS

1. (c) : $\cos^{-1}(\cos(-5)) = \cos^{-1}(\cos(2\pi - 5)) = 2\pi - 5$

$\sin^{-1}(\sin(6)) = \sin^{-1}[\sin(-(2\pi - 6))] = 6 - 2\pi$

$\tan^{-1}(\tan(12)) = \tan^{-1}(\tan(12 - 4\pi)) = 12 - 4\pi$

Now, $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$= 2\pi - 5 + 6 - 2\pi - (12 - 4\pi) = 4\pi - 11$

2. (b) : $S_n = 1(n - 1) + 2(n - 2) + 3(n - 3) + \ldots\ldots + (n - 1)\cdot 1$

Let $t_k = k(n - k) = kn - k^2$

$$S_n = \sum_{k=1}^{n-1} t_k = n\sum_{k=1}^{n-1} k - \sum_{k=1}^{n-1} k^2$$

$$= \frac{n(n-1)n}{2} - \frac{(n-1)(n)\cdot(2n-1)}{6}$$

$$= \frac{n(n-1)}{2}\left[n - \frac{2n-1}{3}\right] = \frac{n(n-1)(n+1)}{6}$$

$$\therefore \frac{S_n}{n!} = \frac{n(n-1)(n+1)}{6\times n!} = \frac{(n+1)}{6\cdot(n-2)!}$$

$$\Rightarrow \frac{2S_n}{n!} = \frac{n+1}{3(n-2)!} = \frac{n-2+3}{3(n-2)!} = \frac{1}{3}\frac{1}{(n-3)!} + \frac{1}{(n-2)!}$$

i.e., $$\frac{2S_n}{n!} - \frac{1}{(n-2)!} = \frac{1}{3(n-3)!}$$

$$\therefore \sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right) = \frac{1}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!}$$

$$= \frac{1}{3}\left[\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}\ldots\ldots\infty\right] = \frac{1}{3}(e-1)$$

3. (a)

4. (b) : Consider $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

$$= -2\times\sin\frac{3\pi}{4}\sin(x) = -\sqrt{2}\sin x$$

Now, $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = 2\times\cos\left(\frac{\pi}{4}\right)\cos x$

$= \sqrt{2}\cos x$

$$\therefore \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

$= \sqrt{2}(\cos x - \sin x)$

Since, $-2 \le \sqrt{2}\,(\cos x - \sin x) \le 2$

$\Rightarrow \quad 1 \le 3 + \sqrt{2}(\cos x - \sin x) \le 5$

$$\Rightarrow 1 \le 3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right) \le 5$$

$$\Rightarrow 0 \le \log_{\sqrt{5}}\left[3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right] \le 2$$

$\therefore$ Range of the function is $[0, 2]$

5. (c) : Let $$L = \lim_{x\to\frac{\pi}{4}} \frac{\frac{\pi}{4}\displaystyle\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$\Rightarrow L = \lim_{x\to\frac{\pi}{4}} \frac{\frac{\pi}{4}\times f(\sec^2 x)\times 2\sec^2 x\tan x}{2x}$$

[By L′ Hospital rule]

$$= \frac{\frac{\pi}{4}f(2)\times 2\times 2}{2\times\frac{\pi}{4}} = 2f(2)$$

6. (b) : We have, $x^2 dy + \left(y - \frac{1}{x}\right)dx = 0$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$$

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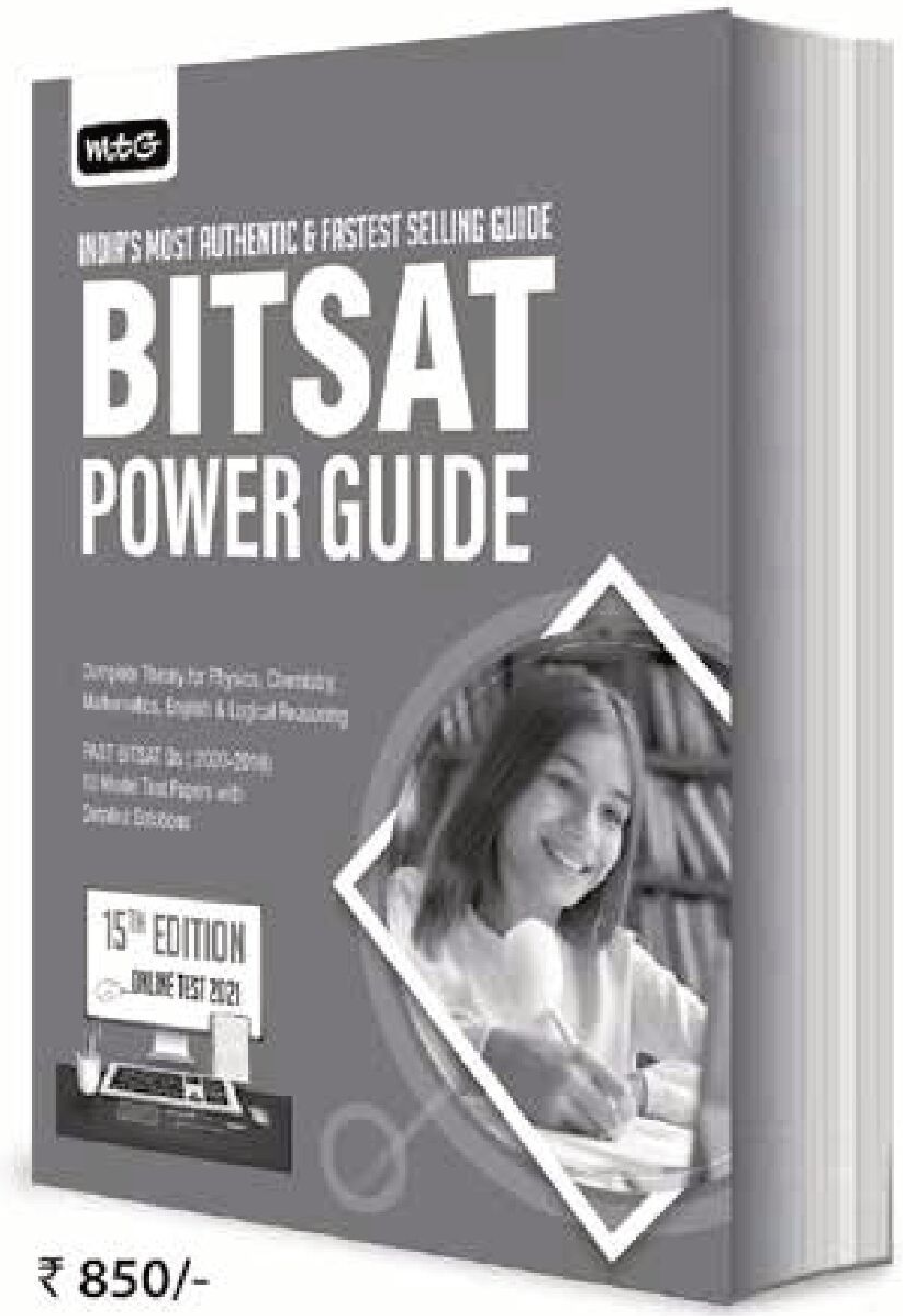

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$$\text{I.F.} = e^{\int \frac{1}{x^2}dx} = e^{\frac{-1}{x}}$$

Thus, the solution is given by

$$y \cdot e^{\frac{-1}{x}} = \int \frac{e^{\frac{-1}{x}}}{x \cdot x^2}dx + C \qquad \text{... (i)}$$

Put $\frac{-1}{x} = t \Rightarrow dt = \frac{dx}{x^2}$

$$\therefore \int \frac{e^{\frac{-1}{x}}}{x^3}dx = -\int te^t\,dt = -(te^t - e^t) + C = e^{\frac{-1}{x}} + \frac{e^{\frac{-1}{x}}}{x} + C$$

$\therefore$ From (i), $ye^{\frac{-1}{x}} = e^{\frac{-1}{x}}\left(1+\frac{1}{x}\right) + C$

Now, $y(1) = 1$

$\Rightarrow 1e^{-1} = e^{-1} \times 2 + C \Rightarrow C = -e^{-1}$

$$\Rightarrow y = \left(1+\frac{1}{x}\right) - \frac{e^{\frac{1}{x}}}{e} \Rightarrow y\left(\frac{1}{2}\right) = 3 - \frac{e^2}{e} = 3 - e$$

7. (c) : Length of perpendicular from vertex to directrix $= a = \frac{1}{4}$

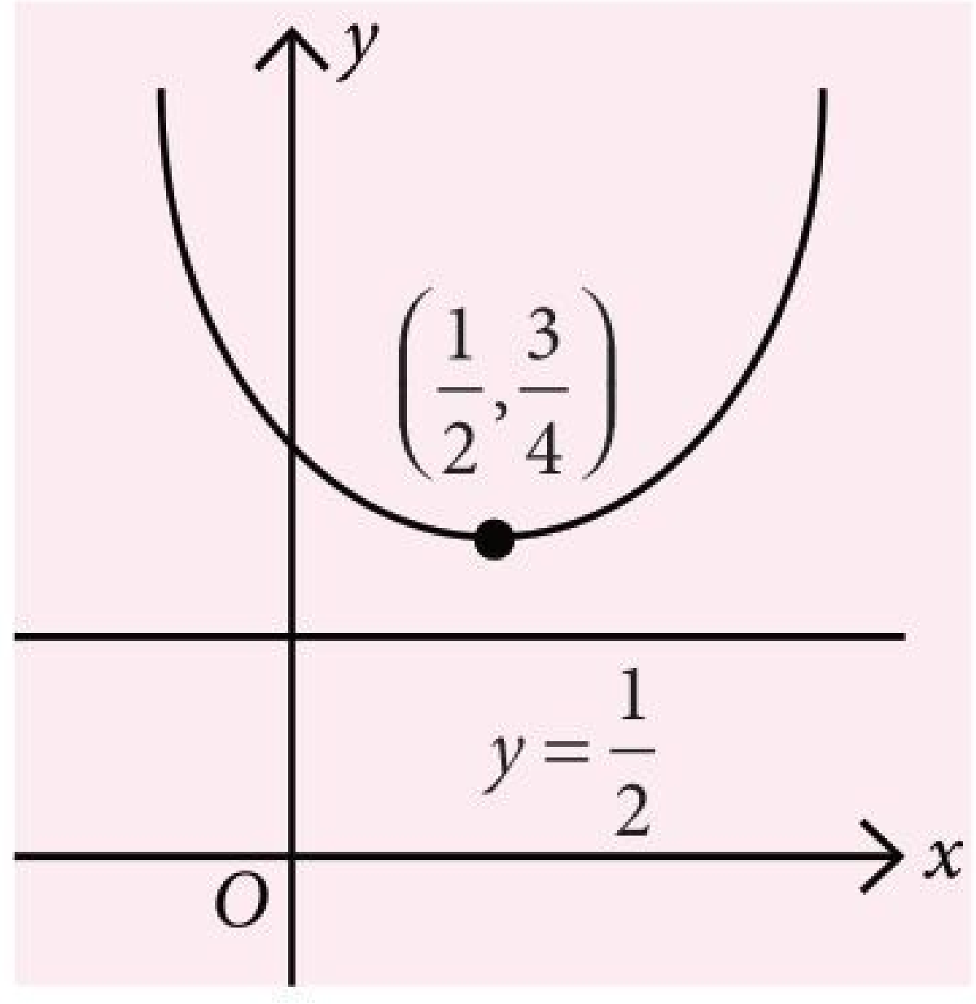
Equation of parabola, is $y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$ ...(i)

$$\Rightarrow \frac{dy}{dx} = 2\left(x - \frac{1}{2}\right) \Rightarrow \left.\frac{dy}{dx}\right|_{\text{At } x = \frac{-1}{2}} = -2$$

$\therefore$ Slope of normal $\left(\text{at } x = \frac{-1}{2}\right) = \frac{1}{2}$

When $x = \frac{-1}{2}$,

$$y - \frac{3}{4} = \left(\frac{-1}{2} - \frac{1}{2}\right)^2 \Rightarrow y = \frac{7}{4}$$



Equation of normal at $\left(\frac{-1}{2}, \frac{7}{4}\right)$ with slope $\frac{1}{2}$ is,

$$y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$$

$\Rightarrow 4y - 7 = 2x + 1$

$$\Rightarrow y = \left(\frac{x}{2} + 2\right) \qquad \text{...(ii)}$$

$\therefore$ For point of intersection,

$$\frac{x}{2} + 2 - \frac{3}{4} = x^2 - x + \frac{1}{4} \qquad \text{(using (i))}$$

$$\Rightarrow x^2 - \frac{3x}{2} - 1 = 0$$

Let x_1, x_2 be roots of it, then $x_1 + x_2 = \frac{3}{2}$ and $x_1x_2 = -1$

$$\Rightarrow (x_1 - x_2)^2 = \frac{9}{4} + 4 = \frac{25}{4}$$

By using (ii), we have

$$y_1 = \frac{x_1}{2} + 2,\ y_2 = \frac{x_2}{2} + 2$$

$$\therefore\ (y_1 - y_2)^2 = \frac{(x_1 - x_2)^2}{4} = \frac{25}{16}$$

Thus, $(x_1 - x_2)^2 + (y_1 - y_2)^2 = PQ^2 = \frac{25}{4} + \frac{25}{16} = \frac{125}{16}$

8. (a) : If the system is inconsistent or has infinitely many solutions $\Rightarrow D = 0$

$$\Rightarrow \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix} = 0$$

$\Rightarrow -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a) = 0$

$\Rightarrow -a^2 + 7a - 12 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow (a - 3)(a - 4) = 0$

$\Rightarrow a = 3$ or 4.

$$\text{Now, } D_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$$

$= -1(-a - 35) + 2(-2 + 7a)$

$= a + 35 - 4 + 14a = 15a + 31$

$\Rightarrow D_1 \neq 0$ for $a = 3$ or 4

Thus, there does not exist any value for which the system has infinitely many solutions.

$\therefore\ n(S_2) = 0$

Since, $D_1 \neq 0$ for $a = 3$ or 4

$\therefore$ For $a = 3$ or 4, the system of equations is inconsistent

i.e., $n(S_1) = 2$

9. (b) : We have, $3y - 2z - 1 = 0 \Rightarrow z = \frac{3y - 1}{2}$

and $3x - z + 4 = 0 \Rightarrow z = 3x + 4$

$\therefore$ Equation of the line is

$$\frac{3x+4}{1} = \frac{3y-1}{2} = \frac{z}{1} \Rightarrow \frac{x + \frac{4}{3}}{\frac{1}{3}} = \frac{y - \frac{1}{3}}{\frac{2}{3}} = \frac{z}{1}$$

$$\Rightarrow \frac{x + \frac{4}{3}}{1} = \frac{y - \frac{1}{3}}{2} = \frac{z}{3} = \lambda \text{ (say)}$$

$\therefore$ Any point on this line is $\left(\lambda - \frac{4}{3}, 2\lambda + \frac{1}{3}, 3\lambda\right)$

Let $\left(\left(\lambda - \frac{4}{3}\right), \left(2\lambda + \frac{1}{3}\right), 3\lambda\right)$ be the foot of perpendicular from (2, −1, 6) to the line.

Now, vector ⊥ to the line is

$$\left(\lambda - \frac{10}{3}\right)\hat{i} + \left(2\lambda + \frac{4}{3}\right)\hat{j} + (3\lambda - 6)\hat{k}$$

$$\Rightarrow \left(\left(\lambda - \frac{10}{3}\right)\hat{i} + \left(2\lambda + \frac{4}{3}\right)\hat{j} + (3\lambda - 6\hat{k})\right)\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \lambda - \frac{10}{3} + 4\lambda + \frac{8}{3} + 9\lambda - 18 = 0$$

$$\Rightarrow 14\lambda = 18 + \frac{2}{3} \Rightarrow \lambda = \frac{4}{3}$$

$\Rightarrow$ Foot of perpendicular ≡ (0, 3, 4)

$\therefore$ Required distance

$$= \sqrt{(2-0)^2 + (-1-3)^2 + (6-4)^2} = \sqrt{4+16+4} = 2\sqrt{6}$$

10. (b) : We have, $y = \sin x + \cos x$

$$\Rightarrow y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

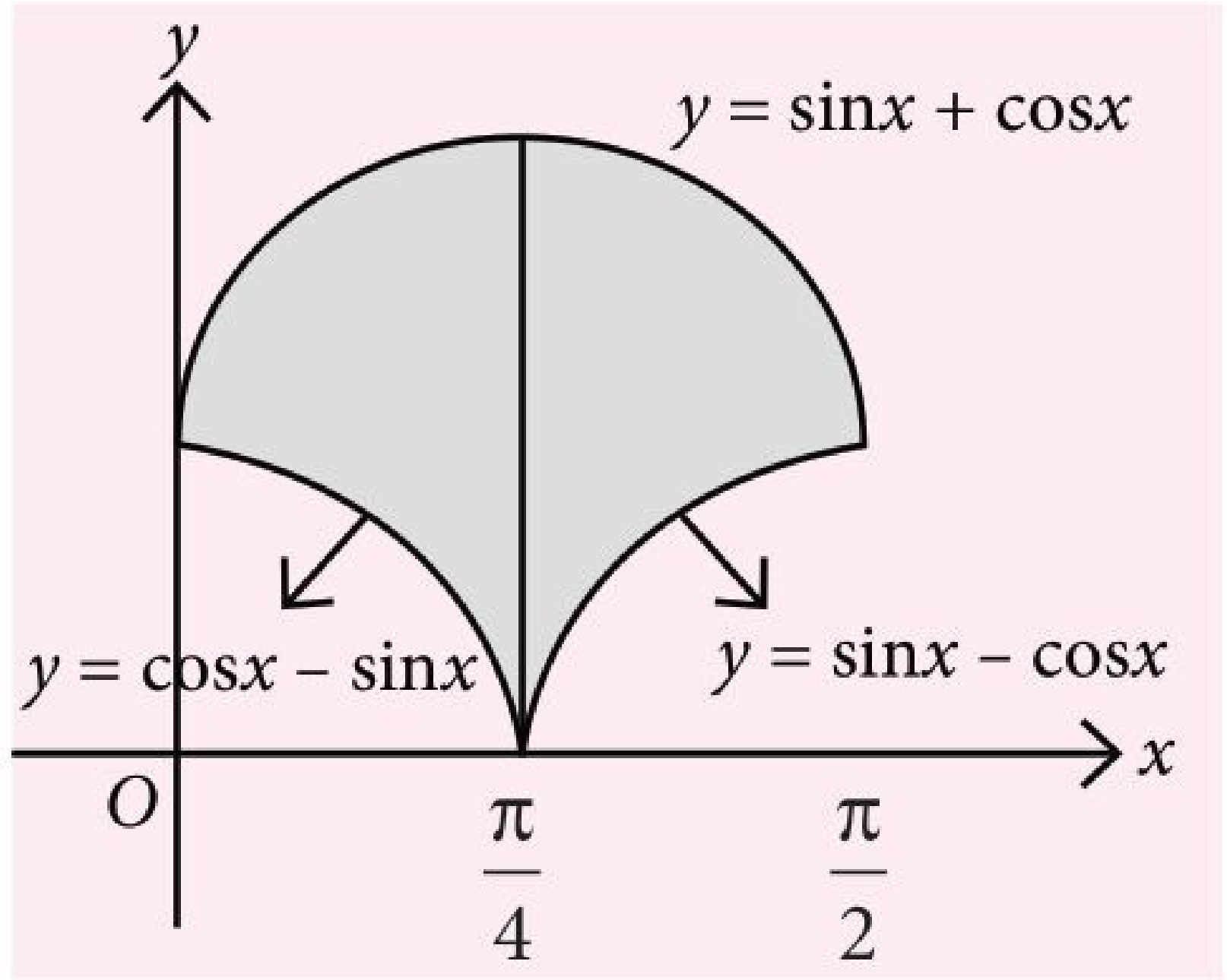
and $y = |\cos x - \sin x|$

$$\Rightarrow y = \cos x - \sin x, \text{ if } x \in \left(0, \frac{\pi}{4}\right)$$

$$= \sqrt{2}\cos\left(x + \frac{\pi}{4}\right), \text{ if } x \in \left(0, \frac{\pi}{4}\right)$$

$$\text{and } y = \sin x - \cos x, \text{ if } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$= -\sqrt{2}\cos\left(x + \frac{\pi}{4}\right), \text{ if } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$



$$\therefore \text{ Required Area} = 2 \times \int_0^{\pi/4} [\sin x + \cos x - (\cos x - \sin x)]dx$$

$$= 4\int_0^{\frac{\pi}{4}} \sin x\, dx = -4[\cos x]_0^{\pi/4} = -4\left(\frac{1}{\sqrt{2}} - 1\right) = 2\sqrt{2}(\sqrt{2} - 1)$$

11. (b) : We have, $2\cos x\left[4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right] = 1$

$$\Rightarrow 2\cos x\left[4\frac{(\cos x + \sin x)}{\sqrt{2}}\frac{(\cos x - \sin x)}{\sqrt{2}} - 1\right] = 1$$

$$\Rightarrow 2\cos x[2\cos 2x - 1] = 1$$

$$\Rightarrow 2\cos x[4\cos^2 x - 3] = 1$$

$$\Rightarrow 4\cos^3 x - 3\cos x = \frac{1}{2}$$

$$\Rightarrow \cos 3x = \frac{1}{2} \Rightarrow 3x = 2n\pi \pm \frac{\pi}{3}$$

Now, putting $n = 1$, we get

$$x = \frac{2\pi}{3} \pm \frac{\pi}{9} \Rightarrow x = \frac{2\pi}{3} - \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}$$

and putting $n = 0$, we get

$$x = \pm\frac{\pi}{9} \Rightarrow x = \frac{\pi}{9} \qquad [\because x \in (0, \pi)]$$

$\therefore$ Number of solutions = 3

Thus, sum of solutions, $S = \frac{2\pi}{3} - \frac{\pi}{9} + \frac{2\pi}{3} + \frac{\pi}{9} + \frac{\pi}{9}$

$$= \frac{4\pi}{3} + \frac{\pi}{9} = \frac{13\pi}{9} \quad \therefore \quad (n, S) \equiv \left(3, \frac{13\pi}{9}\right)$$

12. (c) : For S_1 : $f(2) = 0 \Rightarrow 2a + b = 16$ ...(i)

and $f(4) = 0 \Rightarrow 4a + b = 32$ ...(ii)

Solving (i) and (ii), we get

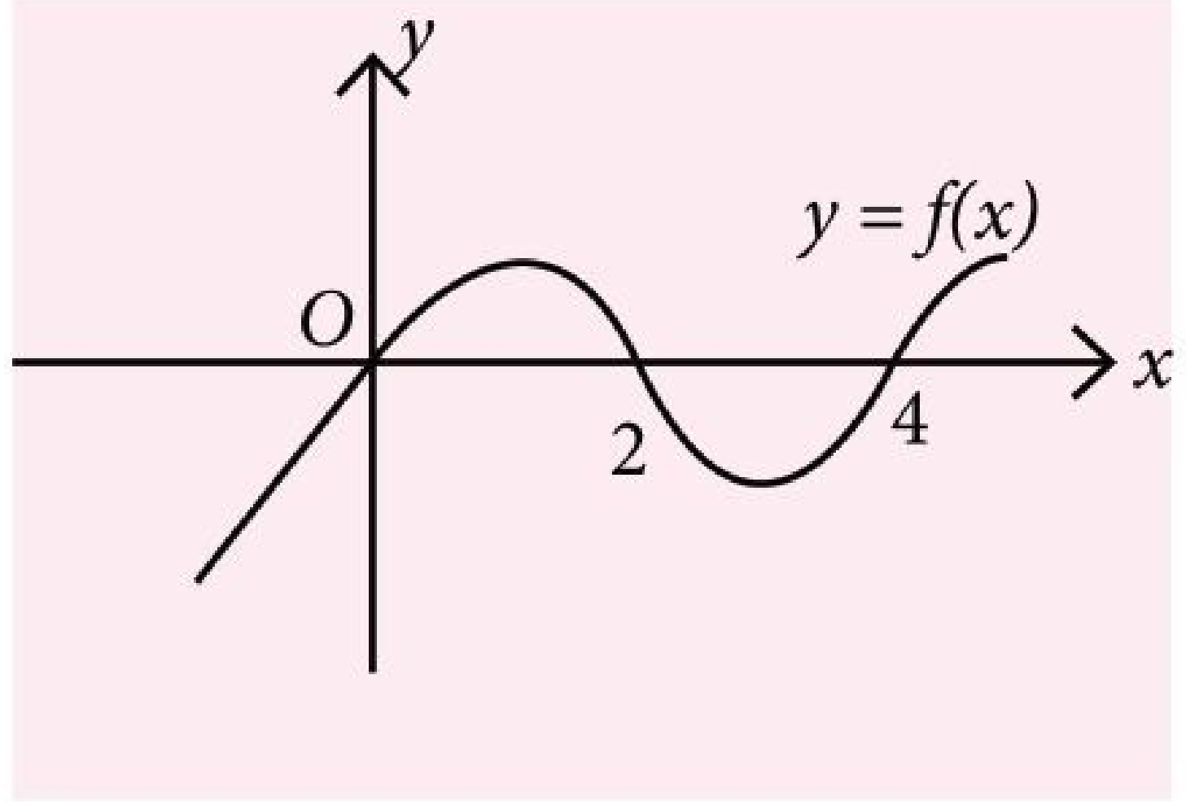
$a = 8$, $b = 0$

$\therefore$ $f(x) = x^3 - 6x^2 + 8x$

$= x(x - 2)(x - 4)$ ...(iii)

Now, $f'(x_2)$

$= 3x_2^2 - 12x_2 + 8$



$$f'(x_2) = 0 \Rightarrow x_2 = 2 \pm \frac{2}{\sqrt{3}}$$

$$\therefore x_2 = 2 + \frac{2}{\sqrt{3}} \in (2, 4)$$

Also, $f'(x_1) = -1 \Rightarrow 3x_1^2 - 12x_1 + 9 = 0$

$\Rightarrow$ $x_1 = 1, 3$ but $x_1 = 3 \in (2, 4) \Rightarrow x_1 < x_2$

$\therefore$ S_1 is true

Since, f is decreasing in $(2, x_4)$ and increasing in $(x_4, 4)$, so, x_4 is a critical point *i.e.*, $f'(x_4) = 0$

$$\Rightarrow x_4 = 2 + \frac{2}{\sqrt{3}} \text{ and } 2f'(x_3) = \sqrt{3}\, f(x_4)$$

$$\Rightarrow 2(3x_3^2 - 12x_3 + 8) = \sqrt{3}\left(2 + \frac{2}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)\cdot\left(\frac{2}{\sqrt{3}} - 2\right)$$

$$\Rightarrow x_3 = \frac{8}{3}, \frac{4}{3} \qquad \text{(Using (iii))}$$

But $2 < x_3 < 4$, so $x_3 = \frac{8}{3} < x_4$. Thus, S_2 is true.

13. (d) : $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix}$

$\Rightarrow$ $A = a_{11}\, a_{22}\, a_{33}$

$\Rightarrow$ $A = (J_{7,3} - J_{4,3})\cdot(J_{8,3} - J_{5,3})\cdot(J_{9,3} - J_{6,3})$

$$= \int_0^{\frac{1}{2}} \frac{x^7 - x^4}{x^3 - 1}dx \cdot \int_0^{\frac{1}{2}} \frac{x^8 - x^5}{x^3 - 1}dx \cdot \int_0^{\frac{1}{2}} \frac{x^9 - x^6}{x^3 - 1}dx$$

$$= \int_0^{\frac{1}{2}} x^4 dx \cdot \int_0^{\frac{1}{2}} x^5 dx \cdot \int_0^{\frac{1}{2}} x^6 dx$$

$$=\left[\frac{x^5}{5}\right]_0^{1/2}\times\left[\frac{x^6}{6}\right]_0^{1/2}\times\left[\frac{x^7}{7}\right]_0^{1/2}=\frac{1}{210\times 2^{18}}$$

Now, $|\text{adj }(A^{-1})| = |A^{-1}|^2$

$$=\frac{1}{|A|^2}=(210\times 2^{18})^2=105^2\times 2^{38}$$

14. (b) : Equation of plane bisecting the angle between the two given planes is

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right|=\left|\frac{2x-3y-6z+1}{\sqrt{4+9+36}}\right|$$

$$\Rightarrow \quad \frac{x-2y-2z+1}{3}=\pm\frac{(2x-3y-6z+1)}{7}$$

$\because \quad a_1a_2 + b_1b_2 + c_1c_2 = 2 + 6 + 12 = 20 > 0$

Therefore, the acute angle bisector is

$7x - 14y - 14z + 7 = -6x + 9y + 18z - 3$

($\because$ –ve sign will give acute bisector)

$\Rightarrow \quad 13x - 23y - 32z + 10 = 0$

So, clearly, $\left(-2,0,\frac{-1}{2}\right)$ lies on it.

15. (d) : Total ways $={}^{64}C_2=63\times 32$

and favourable outcomes = a rectangle is of size 2 × 1 or 1 × 2 = 2 × (7 × 8)

$\therefore \quad$ Required probability $=\frac{2\times 7\times 8}{63\times 32}=\frac{1}{18}$

16. (c) : $p\wedge \sim q = \sim(\sim p \vee q) = \sim(p \to q)$

17. (c) : We have, $\frac{x^2}{9}+\frac{y^2}{1}=1$ ...(i)

and $x^2 + y^2 = 3$ ...(ii)

From (ii), $y^2 = 3 - x^2$ ...(iii)

$\therefore \quad \frac{x^2}{9}+\frac{3-x^2}{1}=1$ (using (i))

$\Rightarrow 2=\frac{8}{9}x^2 \Rightarrow x=\pm\frac{3}{2}\Rightarrow x=\frac{3}{2}$ as $x=\frac{-3}{2}$ (rejected)

From (iii), we get $y^2=3-\frac{9}{4}=\frac{3}{4}$

$\Rightarrow y=\pm\frac{\sqrt{3}}{2} \Rightarrow y=\frac{\sqrt{3}}{2}$ as $y=\frac{-\sqrt{3}}{2}$ (rejected)

$\therefore \quad$ Point of intersection of the two curves is $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

[As point lies in the first quadrant]

Thus, tangent to ellipse at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is

$\frac{x}{6}+\frac{y\sqrt{3}}{2}=1 \Rightarrow$ Slope $=m_1=\frac{-1}{3\sqrt{3}}$

Also, tangent to circle at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is $\frac{3x}{2}+\frac{\sqrt{3}y}{2}=1$

$\Rightarrow \quad$ Slope $= m_2=-\sqrt{3}$

$$\text{Now, } \tan\theta=\left|\frac{-\sqrt{3}+\frac{1}{3\sqrt{3}}}{1+\frac{1}{3}}\right|=\frac{8}{3\sqrt{3}\times\frac{4}{3}}=\frac{2}{\sqrt{3}}$$

18. (d) : We have, $f(x)=x+\int_0^{\pi/2}\sin x\cos y\cdot f(y)dy.$

Let $\int_0^{\pi/2}\cos y\, f(y)\,dy=K$ ...(i)

$\therefore \quad f(x) = x + K\sin x$

$\Rightarrow \quad \cos x\, f(x) = x \cos x + K \sin x \cos x$

(Multiplying cos$x$ to both sides)

$$\Rightarrow \quad \int_0^{\pi/2}\cos x\, f(x)dx=\int_0^{\pi/2}x\cos x\, dx+K\int_0^{\pi/2}\sin x\cos x\, dx$$

$$\Rightarrow \quad K=\left[x\sin x+\cos x\right]_0^{\pi/2}+\frac{K}{2}\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} \quad \text{(Using (i))}$$

$$\Rightarrow \quad K=\left(\frac{\pi}{2}+0\right)-1+\frac{K}{2}$$

$$\Rightarrow \quad \frac{K}{2}=\frac{\pi}{2}-1\Rightarrow K=(\pi-2)$$

$\therefore \quad f(x) = x + K\sin x = x + (\pi - 2) \sin x$

19. (d) : Total number of triangles = ${}^{15}C_3$ = 455

When $i + j + k = 15$, then

(i, j, k) can be (1, 2, 12), (1, 3, 11), (1, 4, 10), (1, 5, 9), (1, 6, 8), (2, 3, 10), (2, 4, 9), (2, 5, 8), (2, 6, 7), (3, 4, 8), (3, 5, 7), (4, 5, 6)

$\therefore \quad$ Number of triangles with $i + j + k = 15$ is 12.

$\Rightarrow \quad$ Required number of triangles = 455 – 12 = 443

20. (d) : Let $t_n=\frac{1}{a_n\cdot a_{n+1}}=\frac{1}{d}\frac{(a_{n+1}-a_n)}{a_{n+1}\,a_n}$

$$\Rightarrow t_n=\frac{1}{d}\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)$$

$$\Rightarrow \sum_{n=1}^{20}t_n=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}...+\frac{1}{a_{20}}-\frac{1}{a_{21}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{21}}\right)=\frac{(a_{21}-a_1)}{da_1\,a_{21}}=\frac{20d}{d\,a_{21}\cdot a_1}=\frac{20}{a_1a_{21}}=\frac{4}{9}$$

$\therefore \quad a_1a_{21} = 45$ ...(i)

$\Rightarrow \quad 189=\frac{21}{2}(a_1+a_{21})$

$\Rightarrow \quad a_1 + a_{21} = 18$ (Using (i))

$\Rightarrow \quad a_1 + \dfrac{45}{a_1} = 18 \quad \Rightarrow \quad a_1^2 - 18a_1 + 45 = 0$

$\Rightarrow \quad (a_1 - 15)(a_1 - 3) = 0 \;\Rightarrow\; a_1 = 3$ or 15

Case 1 : When $a_1 = 3$

$\Rightarrow \quad a_{21} = 15 = 3 + 20d \;\Rightarrow\; d = \dfrac{3}{5}$

$\therefore \quad a_6 = 3 + 5 \times \dfrac{3}{5} = 6$

and $a_{16} = 3 + 15 \times \dfrac{3}{5} = 12$

$\therefore \quad a_6\, a_{16} = 72$

Case 2 : When $a_1 = 15$

$\Rightarrow \quad a_{21} = 3 = 15 + 20d \Rightarrow d = \dfrac{-3}{5}$

$\therefore \quad a_6 = 15 + 5 \times \left(\dfrac{-3}{5}\right) = 12$

and $a_{16} = 15 + 15 \times \left(\dfrac{-3}{5}\right) = 6$

$\therefore \quad a_6\, a_{16} = 72$

21. (6) : The points of intersection of the 3 given lines are (1, 2), (7, 5) & (2, 3).

Let $(x_1\, y_1), (x_2, y_2)$ & (x_3, y_3) be the vertices of triangle ABC.

Now, $x_1 + x_2 = 2,\; x_2 + x_3 = 14,\; x_3 + x_1 = 4$

$x_1 + x_2 + x_3 = 10 \Rightarrow x_3 = 8,\; x_2 = 6,\; x_1 = -4$

Also, $y_1 + y_2 = 4,\; y_2 + y_3 = 10,\; y_3 + y_1 = 6$

$y_1 + y_2 + y_3 = 10 \Rightarrow y_1 = 0,\; y_2 = 4,\; y_3 = 6$

Hence, the vertices of triangle are (–4, 0), (6, 4) & (8, 6)

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} -4 & 0 & 1 \\ 6 & 4 & 1 \\ 8 & 6 & 1 \end{vmatrix}$$

$= \dfrac{1}{2}(4 \times 2 + 36 - 32) = 6$

22. (26) : Let $xf(x) + 2 = \lambda\,(x - 2)\,(x - 3)(x - 4)\,(x - 5)$ ...(i)

Putting $x = 0$ in (i), we get

$2 = 120\,\lambda \Rightarrow \lambda = \dfrac{1}{60}$

$x\,f(x) + 2 = \dfrac{1}{60}(x-2)(x-3)(x-4)(x-5)$ ...(ii)

Putting $x = 10$ in (ii), we get

$10\,f(10) + 2 = \dfrac{1}{60} \times 8 \times 7 \times 6 \times 5 = 28$

$\Rightarrow \quad 10\,f(10) = 26$

$\therefore \quad 52 - 10\,f(10) = 26$

23. (2) : For $-2 < x < -1$,

$f(x) = -2 \times (x^2 - 1) + \sin(\pi) - (-2 + 1) = (3 - 2x^2)$

For $-1 \le x < 0$,

$f(x) = -1 \times (1 - x^2) + \sin\left(\dfrac{\pi}{2}\right) - (-1 + 1) = 1 - (1 - x^2) = x^2$

For $0 \le x < 1$,

$f(x) = 0 \cdot |x^2 - 1| + \sin\left(\dfrac{\pi}{3}\right) - (0 + 1) = \sin\dfrac{\pi}{3} - 1 = \dfrac{\sqrt{3}}{2} - 1$

For $1 \le x < 2$,

$f(x) = (x^2 - 1) + \sin\left(\dfrac{\pi}{1+3}\right) - (1 + 1)$

$= x^2 - 1 + \dfrac{1}{\sqrt{2}} - 2$

$$\therefore \quad f(x) = \begin{cases} 3 - 2x^2 & , \quad x \in (-2, -1) \\ x^2 & , \quad x \in [-1, 0) \\ \dfrac{\sqrt{3}}{2} - 1 & , \quad x \in [0, 1) \\ \dfrac{1}{\sqrt{2}} - 3 + x^2, & \quad x \in [1, 2) \end{cases}$$

Now, $\lim\limits_{x \to -1^-} f(x) = 1 = f(-1)$ & $\lim\limits_{x \to -1^+} f(x) = 1$

$\therefore \quad f(x)$ is continuous at $x = -1$.

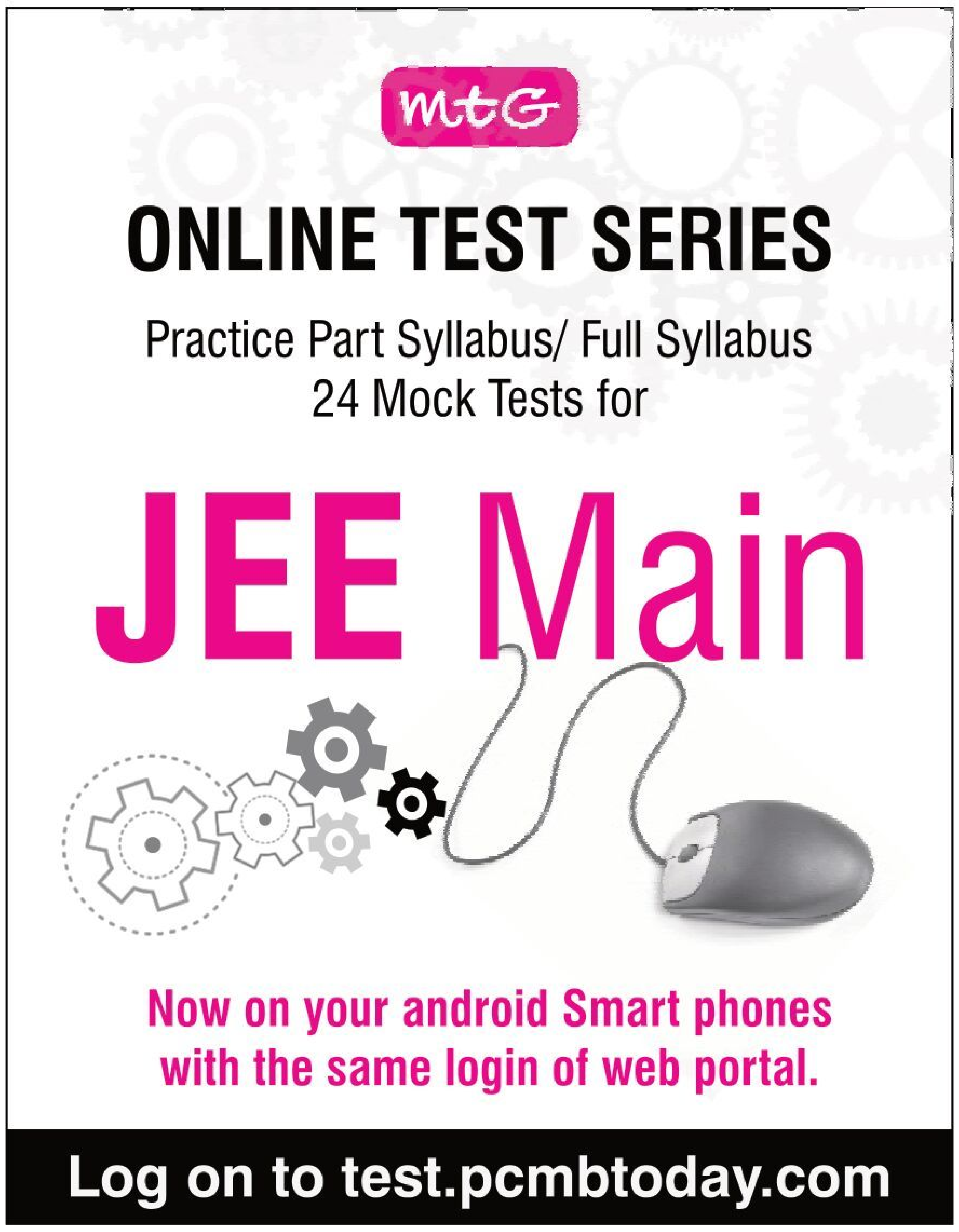
$\lim\limits_{x \to 0^-} f(x) = 0$ & $\lim\limits_{x \to 0^+} f(x) = \dfrac{\sqrt{3}}{2} - 1$

$\therefore \quad f(x)$ is discontinuous at $x = 0$.

$\lim\limits_{x \to 1^-} f(x) = \dfrac{\sqrt{3}}{2} - 1$ and $\lim\limits_{x \to 1^+} f(x) = \dfrac{1}{\sqrt{2}} - 2$

$\therefore \quad f(x)$ is discontinuous at $x = 1$.

$\Rightarrow \quad f(x)$ is discontinuous at 2 points.



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24. (77) : Let us first count the number of total words that appear before FARMER.

A _ _ _ _ _ $\rightarrow \frac{5!}{2!}=60$

E _ _ _ _ _ $\rightarrow \frac{5!}{2!}=60$

F A E _ _ _ $\rightarrow \frac{3!}{2!}=3$

F A M _ _ _ $\rightarrow \frac{3!}{2!}=3$

F A R E _ _ $\rightarrow 2$

$\therefore$ Total number of numbers appearing before FARMER is 128.

Let us calculate the number of words appearing before FARMER that have 2R together.

A _ _ _ _ _ $\rightarrow 4! = 24$

E _ _ _ _ _ $\rightarrow 4! = 24$

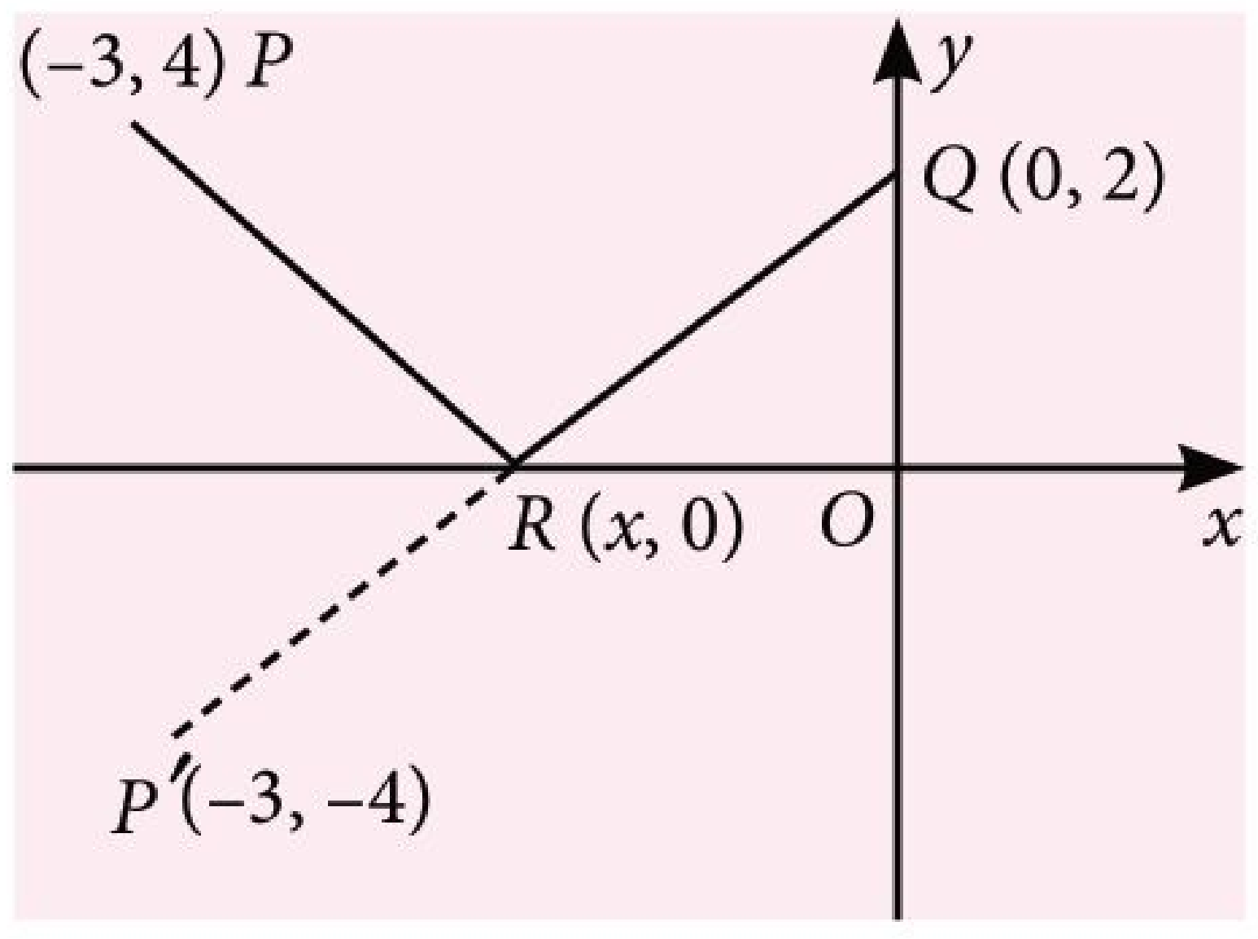
F A E _ _ _ $\rightarrow 2! = 2$

F A M _ _ _ $\rightarrow 2! = 2$

$\therefore$ Total words = 52

No. of words that appear before FARMER that have 2R not appearing together = 128 – 52 = 76

$\therefore$ Serial number of FARMER in the list is 77.



25. (1250) : Let P' be the mirror image of P about x-axis.

$\therefore \quad PR = P'R$

Now, $PR + RQ$

$= P'R + RQ$

$P'R + RQ$ is minimum if P', R, Q are collinear.

$\Rightarrow$ Slope of $P'R$ = slope of RQ

$$\Rightarrow \frac{-4-0}{-3-x}=\frac{2-0}{0-x} \Rightarrow 4x = -6 - 2x$$

$\Rightarrow \quad x = -1 \Rightarrow R = (-1, 0)$

$\therefore \quad PR^2 + RQ^2 = P'R^2 + RQ^2$

$= (-3 + 1)^2 + (-4)^2 + 1^2 + 2^2$

$= 4 + 16 + 5 = 25$

$\therefore \quad 50 \times (PR^2 + RQ^2) = 50 \times 25 = 1250$

26. (7) : $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$

$\Rightarrow \quad f(1) = 9$

$f'(x) = 6x^5 + 8x^3 + 3x^2 + 2 \Rightarrow f'(1) = 19$

$$\text{Now, } \lim_{x\to 1}\frac{x^n f(1)-f(x)}{x-1} \qquad \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 1}\frac{nx^{n-1} f(1)-f'(x)}{1} \qquad \text{[By L}'\text{ Hospital rule]}$$

$= n \times 9 - f'(1) = 9n - 19 = 44$ (Given)

$\Rightarrow \quad 9n = 63 \Rightarrow n = 7$

27. (781) : $\bar{X}=\sum x_i\, p_i = \frac{-2}{5}-a+1+\frac{4}{5}+6b=2.3$

$\Rightarrow \quad 6b - a = 2.3 - 1 - 0.4 = 0.9 = \frac{9}{10}$ ... (i)

Also, $\Sigma p(x_i) = 1$

$\Rightarrow \frac{1}{5}+a+\frac{1}{3}+\frac{1}{5}+b=1 \Rightarrow a+b=\frac{2}{3}-\frac{2}{5}=\frac{4}{15}$ ... (ii)

Adding (i) and (ii), we have $7b=\frac{9}{10}+\frac{4}{15} \Rightarrow b=\frac{1}{6}$

$\therefore a=\frac{4}{15}-\frac{1}{6}=\frac{1}{10}$

Now, $\sigma^2=\sum p_i\, x_i^2-\bar{X}^2$

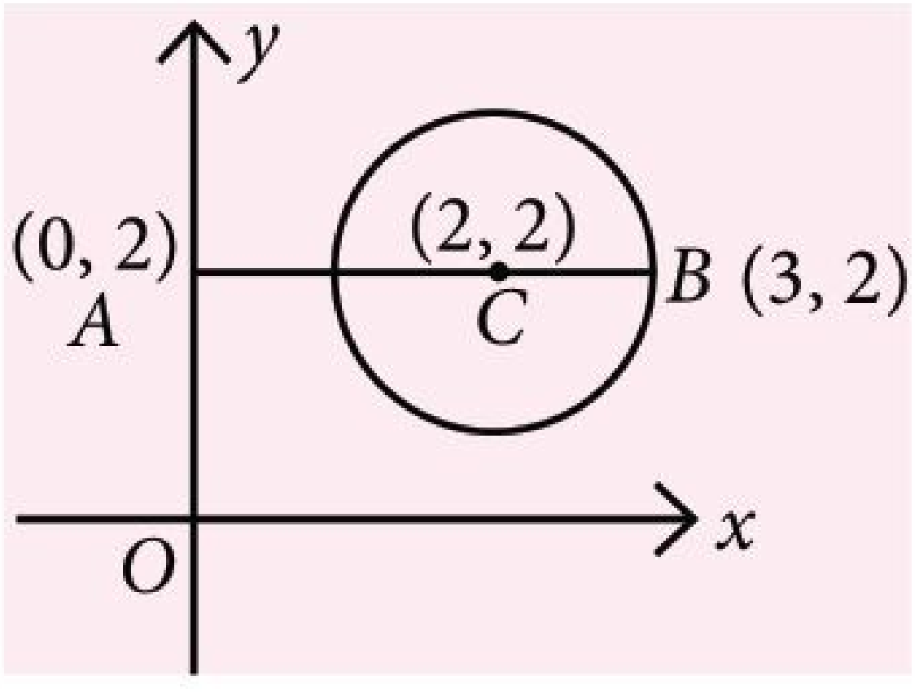
$=\frac{4}{5}+\frac{1}{10}\times 1+\frac{1}{3}\times 9+\frac{1}{5}\times 16+\frac{1}{6}\times 36-(2.3)^2$

$=\frac{4}{5}+\frac{1}{10}+3+\frac{16}{5}+6-(2.3)^2 = 13.1 - 5.29 = 7.81$

$\therefore \quad 100\,\sigma^2 = 781$

28. (5) : Maximum value of $|3iz + 6|$ is equal to maximum value of $3|z - 2i|$.

Maximum value of $3|z - 2i|$ is the maximum distance of points lying on or inside the circle from A, which is attained when z is at B (3, 2).



$\therefore \quad z = 3 + 2i = a + ib \Rightarrow a + b = 5$

29. (924) : Sum of coefficients of $2^n = 4096$

$\Rightarrow \quad 2^n = 2^{12} \Rightarrow n = 12$

$\therefore$ Greatest coefficient $= {}^{12}C_6 = 924$

30. (1494) : Let $\vec{v}=\lambda\vec{a}+\mu\vec{b}$

$=\lambda(2\hat{i}-\hat{j}+2\hat{k})+\mu(\hat{i}+2\hat{j}-\hat{k})$

$\Rightarrow \quad \vec{v}=(2\lambda+\mu)\hat{i}+(2\mu-\lambda)\hat{j}+(2\lambda-\mu)\hat{k}$

Now, $\vec{v}\cdot(3\hat{i}+2\hat{j}-\hat{k})=0$

$\Rightarrow \quad 3(2\lambda + \mu) + 2(2\mu - \lambda) + \mu - 2\lambda = 0$

$\Rightarrow \quad 2\lambda + 8\mu = 0 \Rightarrow \lambda = -4\mu$

We have, $\frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}=19$

$$\Rightarrow \frac{\left[(2\lambda+\mu)\hat{i}+(2\mu-\lambda)\hat{j}+(2\lambda-\mu)\hat{k}\right]\cdot(2\hat{i}-\hat{j}+2\hat{k})}{3}=19$$

$\Rightarrow \quad 57 = 4\lambda + 2\mu + \lambda - 2\mu + 4\lambda - 2\mu$

$\Rightarrow \quad 9\lambda - 2\mu = 57 \Rightarrow -38\mu = 57$

$\Rightarrow \mu=\frac{-3}{2}$ and $\lambda = 6$

$\therefore \quad \vec{v}=\left(12-\frac{3}{2}\right)\hat{i}-9\hat{j}+\left(12+\frac{3}{2}\right)\hat{k}$

$\Rightarrow \quad 2\vec{v}=21\hat{i}-18\hat{j}+27\hat{k}$

$\therefore \quad |2\vec{v}|^2=21^2+(-18)^2+27^2=1494$

◆ ◆

PRACTICE PAPER 2022

JEE MAIN

Time : 1 hr.

Series - 2

The entire syllabus of Mathematics of JEE MAIN is being divided into four parts. Each part contains 4 units. There will be a Practice Paper which will be published in the subsequent issues. The syllabus for module break-up is given below:

Units	Topic	Syllabus In Detail
V	Mathematical Inductions	Principle of mathematical Induction and its simple applications.
VI	Binomial Theorem and its Simple Applications	Binomial theorem for a positive integral index, general term and middle term, properties of Binomial coefficients and simple applications.
VII	Sequence and Series	Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M. and G.M., sum up to *n* terms of special series; S_n, S_{n^2}, S_{n^3}. Arithmetico-Geometric progression.
VIII	Limit, Continuity and Differentiability	Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic and exponential functions, inverse function. Graphs of simple functions. Limits, continuity and differentiability. Differentiation of the sum, difference, product and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite and implicit functions; derivatives of order up to two, Rolle's and Lagrange's Mean value Theorems, Applications of derivatives: Rate of change of quantities, monotonic - Increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normal.

SINGLE OPTION CORRECT TYPE

1. Let $P(n)$ be the statement $n^2 - n + 41$ is prime, then which of the following is not true?

(a) $P(2)$ (b) $P(3)$
(c) $P(41)$ (d) None of these

2. The two consecutive terms in the expansion of $(3 + 2x)^{74}$, whose coefficients are equal, is/are

(a) 30th and 31st terms (b) 29th and 30th terms
(c) 31st and 32nd terms (d) 28th and 29th terms

3. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a+b+c=\frac{3}{2}$. Then $a =$

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$
(c) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

4. $\underset{x \to 1}{\text{Lt}}\ (1-x^2)\log_{x^2} 2$ equals

(a) log 2 (b) log 3 (c) –log 2 (d) –log 3

5. If $f(x) = \begin{cases} \dfrac{(5^x-1)^3}{\sin\left(\dfrac{x}{a}\right)\log\left(1+\dfrac{x^2}{3}\right)}, & x \neq 0 \\ 9(\log_e 5)^3, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of 'a' equals

(a) 3 (b) 2
(c) 1 (d) None of these

6. If f & g are differentiable functions on [0, 1], such that $g(x) \neq 0$ for all $x \in (0, 1)$, $f(0) = 2 = g(1)$ and $f(1) = 6$, $g(0) = 0$, then for some $c \in (0, 1)$

(a) $2f'(c) = 3g'(c)$ (b) $f'(c) = g'(c)$
(c) $f'(c) = 2g'(c)$ (d) $g'(c) = 2f'(c)$

7. If $(1+x)^n = 1+\frac{3}{4}+\frac{3}{4}\cdot\frac{5}{8}+\frac{3}{4}\cdot\frac{5}{8}\cdot\frac{7}{12}+\ldots\infty$, then the set of values of x and n are respectively

(a) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (b) $\left(-\frac{1}{2},-\frac{3}{2}\right)$

(c) $\left(\frac{1}{2},\frac{3}{2}\right)$ (d) None of these

8. If a and b are natural numbers such that $a^2 - b^2$ is prime number, then $a^2 - b^2$ equals

(a) $a+b$ (b) $a-b$ (c) ab (d) 1

9. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then

$\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+\ldots\ldots+n\frac{C_n}{C_{n-1}}$ equals

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

10. The sum of four numbers in A.P. is 48, and the product of the extremes is to the product of the two middle terms is 27 to 35. The largest term of the A. P. is

(a) 10 (b) 12 (c) 14 (d) 18

11. If $\lim\limits_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\frac{k_1}{k_2}$, where k_1 and k_2 are coprime number, then the value of $k_1^2 - k_2^2$ equals

(a) 1 (b) 2 (c) 4 (d) 5

12. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3)$ ... then sum of the numbers in the nth group is

(a) $\frac{n^2}{12}(n^2+1)(n^2+4)$ (b) $\frac{n^3}{8}(n^2+1)(n^2+3)$

(c) $\frac{n^3}{8}(n^2+2)(n^2+4)$ (d) $\frac{n^2}{16}(n^2+1)(n^2+4)$

13. If $2^{2017}+\alpha$ is divisible by 17, then minimum value of α =

(a) 11 (b) 12 (c) 15 (d) 16

14. The value of $\sum\limits_{r=1}^{n} r^2 - \sum\limits_{m=1}^{n}\sum\limits_{r=1}^{m} r$ equals

(a) $\frac{1}{2}\left[\sum\limits_{r=1}^{n} r^2 - \sum\limits_{r=1}^{n} r\right]$ (b) ${}^{n+1}C_2$

(c) ${}^{n+1}C_4$ (d) $\frac{1}{2}\left[\sum\limits_{r=1}^{n} r^2 + \sum\limits_{r=1}^{n} r\right]$

15. The greatest value of positive integer, which divides $(r+2)(r+3)(r+4)(r+5)\ \forall\ r \in N$, is

(a) 4! (b) 2! (c) 5! (d) 3!

16. If the coefficients of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal, then

(a) $a+b=1$ (b) $a-b=1$

(c) $ab=-1$ (d) $ab=1$

17. If $(16)^{15}+2\cdot 17(16)^{14}+3(17)^2(16)^{13}+\ldots\ldots+16(17)^{15}$ $= m(16)^{15}$ then value of $\sqrt{m+33}$ equals

(a) 15 (b) 16 (c) 17 (d) 18

18. $\lim\limits_{n\to\infty}\frac{1^m+2^m+3^m+\ldots+n^m}{n^{m+1}}$ equals

(a) $\frac{1}{m-1}$ (b) $\frac{1}{m}$ (c) $\frac{1}{m+2}$ (d) $\frac{1}{m+1}$

19. Let $f(x)$ be a polynomial of degree 3 such that $f(1)=1, f'(1)=2, f''(1)=3, f'''(1)=4$. Then $f(0)$ =

(a) $\frac{1}{2}$ (b) $\frac{-1}{3}$ (c) $\frac{-1}{6}$ (d) $\frac{1}{6}$

20. If $x = e^t \sin t$, $y = e^t \cos t$, then

$(x+y)^2\frac{d^2y}{dx^2}=\lambda\left(\frac{xdy}{dx}-y\right)$, where λ =

(a) 2 (b) -2 (c) 3 (d) -1

NUMERICAL ANSWER TYPE

21. If $\sum\limits_{i=1}^{25} a_i = 625$ such that $a_1, a_2, a_3, \ldots \in$ A.P., then $\sum\limits_{j=1}^{13} a_{2j-1}$ equals ______.

22. Number of points at which $f(x)=\frac{1}{\log|x|}$ is discontinuous is ______.

23. The number of terms in the expansion of $(a+b+c+d)^{15}$ is, if $a \neq b \neq c \neq d$ ______.

24. Let $P(n) = 3^{2n}\ \forall\ n \in N$, when divided by 8, leaves the remainder ______.

25. The absolute value of coefficient of x^{50} in the expansion of $\frac{(3-5x)}{(1-x)^2}$ equals ______.

26. If one G.M., g and two A.M.'s p and q are inserted between two numbers a and b, then $\left|\dfrac{(2p-q)(p-2q)}{g^2}\right| =$ ______.

27. The value of B, if $\underset{x\to 0}{\text{Lt}} \dfrac{Ae^x - B\cos x + Ce^{-x}}{x\sin x}$ equals 2, is ______.

28. The remainder when $(32)^{(32)^{(32)}}$ is divided by 7, is ______.

29. The sum to infinity of the series

$$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots \text{ is } \_\_\_\_\_\_.$$

30. $\underset{x\to 0}{\text{Lt}} \dfrac{\log(1+x^3)}{\sin^3 x}$ equals ______.

SOLUTIONS

1. **(c) :** $P(n) = n^2 - n + 41$

$P(2) = 2^2 - 2 + 41 = 43$ (Prime, true)

$P(3) = 3^2 - 3 + 41 = 47$ (Prime, true)

$P(41) = 41^2 - 41 + 41 = (41)^2$ (not prime, false)

$\therefore$ $P(41)$ is not true.

2. **(a) :** Let the consecutive terms are T_r and T_{r+1} having equal coefficients.

$\therefore$ Coefficient of T_r^{th} term = Coefficient of T_{r+1}^{th} term

$$\Rightarrow\ {}^{74}C_{r-1}\, 3^{74}\left(\frac{2}{3}\right)^{r-1} = {}^{74}C_r\, 3^{74}\left(\frac{2}{3}\right)^{r}$$

$$\Rightarrow\ {}^{74}C_{r-1}\times 3 = {}^{74}C_r \times 2$$

$$\Rightarrow\ \frac{3\times 74!}{(r-1)!(75-r)!} = \frac{2\times 74!}{r!(74-r)!}$$

$$\Rightarrow\ \frac{3}{75-r} = \frac{2}{r} \Rightarrow 150 - 2r = 3r \Rightarrow r = 30$$

$\therefore$ T_{30} and T_{31} are two consecutive terms, whose coefficients are same.

3. **(d) :** Since, a, b, c are in A.P, therefore $2b = a + c$

$$\therefore\ a+b+c=\frac{3}{2} \Rightarrow 3b = \frac{3}{2} \Rightarrow b = \frac{1}{2}$$

$a = \dfrac{1}{2} - d,\ c = \dfrac{1}{2} + d$, where d is common difference.

Now, a^2, b^2, c^2, are in G.P.

$$\Rightarrow\ a^2c^2 = b^4 = \frac{1}{16} \Rightarrow ac = \pm\frac{1}{4} \Rightarrow \frac{1}{4} - d^2 = \pm\frac{1}{4}$$

If $ac = \dfrac{1}{4}$, $d = 0$; $a = b = c$

and if $ac = -\dfrac{1}{4} \Rightarrow d^2 = \dfrac{1}{2} \Rightarrow d = \dfrac{1}{\sqrt{2}}$

[$\because$ d can't be negative as $a < b < c$]

$$\therefore\ a = \frac{1}{2} - d = \frac{1}{2} - \frac{1}{\sqrt{2}}$$

4. **(c) :** $\underset{x\to 1}{\text{Lt}}\,(1-x^2)\log_{x^2} 2 = \underset{x\to 1}{\text{Lt}} \dfrac{1-x^2}{\log_2 x^2}$ (0/0 form)

$= \underset{x\to 1}{\text{Lt}} \dfrac{1}{2}\cdot\dfrac{d(1-x^2)}{d(\log_2 x)}$ [Using L' Hospital Rule]

$$= \frac{1}{2}\underset{x\to 1}{\text{Lt}} \frac{-2x}{1/\log 2}\times x$$

$= -\log 2$

5. **(a) :**
$$\lim_{x\to 0} \frac{(5^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^3 \frac{x^3}{\left(\frac{x}{a}\right)\dfrac{\sin\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)}} \times \frac{1}{\dfrac{\log\left(1+\frac{x^2}{3}\right)}{\frac{x^2}{3}}\times\dfrac{x^2}{3}}$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^3 \frac{1\times a}{\left(\dfrac{\sin\left(\frac{x}{a}\right)}{\frac{x}{a}}\right)}\cdot\frac{3}{\dfrac{\log\left(1+\frac{x^2}{3}\right)}{\frac{x^2}{3}}}$$

$$\left(\text{Using } \lim_{x\to 0}\frac{\log(1+x)}{x} = 1,\ \lim_{x\to 0}\frac{\sin x}{x} = 1 \text{ and } \lim_{x\to 0}\frac{a^x-1}{x} = \log_e a\right)$$

$= 3a(\log_e 5)^3$

Now, $f(0) = 9(\log_e 5)^3$

Since, $f(x)$ is continuous at $x = 0$

$\therefore$ $9(\log_e 5)^3 = 3a(\log_e 5)^3 \Rightarrow a = 3$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(c)
6.	(b)	**7.**	(a,d)	**8.**	(c,d)	**9.**	(a,b,c,d)		
10.	(a,b,d)	**11.**	(a,b)	**12.**	(c,d)	**13.**	(a)	**14.**	(c)
15.	(d)	**16.**	(a)	**17.**	(7)	**18.**	(9)	**19.**	(1)
20.	(6)								

6. (c) : Given f & g are differentiable functions on [0, 1]. Therefore by cauchy mean value theorem there exist a number $c \in (0, 1)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(1)-f(0)}{g(1)-g(0)} = \frac{6-2}{2-0}$$

$$\Rightarrow \frac{f'(c)}{g'(c)} = \frac{4}{2} \Rightarrow f'(c) = 2g'(c)$$

7. (b): Given, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2} \cdot x^2 + \ldots$

$$= 1 + \frac{3}{4} + \frac{3}{4} \cdot \frac{5}{8} + \frac{3}{4} \cdot \frac{5}{8} \cdot \frac{7}{12} + \ldots\ldots$$

$$\Rightarrow \quad nx = \frac{3}{4} \text{ and } \frac{n(n-1)}{2} x^2 = \frac{3}{4} \cdot \frac{5}{8}$$

$$\therefore \quad \frac{nx(nx-x)}{2} = \frac{3}{4} \cdot \frac{5}{8}$$

$$\Rightarrow \quad \frac{3}{4}\left(\frac{3}{4} - x\right)\frac{1}{2} = \frac{3}{4} \cdot \frac{5}{8} \quad \Rightarrow \quad \left(\frac{3}{4} - x\right)\frac{1}{2} = \frac{5}{8}$$

$$\Rightarrow \quad \frac{3}{4} - x = \frac{5}{4} \quad \Rightarrow \quad \frac{3}{4} - \frac{5}{4} = x \Rightarrow -1/2 = x$$

Now, $nx = \frac{3}{4} \Rightarrow n\left(-\frac{1}{2}\right) = \frac{3}{4} \Rightarrow n = -\frac{3}{2}$

$\therefore$ Set of values of x and n are respectively $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

8. (a) : Consider, $a^2 - b^2 = (a+b)(a-b)$ is a prime (given)

$\therefore$ $a^2 - b^2$ is either divisible by 1 or the number itself.

$\therefore$ $a - b = 1$ and $a^2 - b^2 = a + b$ ($\because a + b \neq 1$ as $a, b \in N$)

$\therefore$ $a^2 - b^2 = a + b$

9. (d) : Consider $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots\ldots + n\frac{C_n}{C_{n-1}}$

$$= \frac{n}{1} + 2 \cdot \frac{n(n-1)}{2 \cdot n} + \frac{3n(n-1)(n-2)}{6} \times \frac{2}{n(n-1)} + \ldots\ldots + n \cdot \frac{1}{n}$$

$= n + (n-1) + (n-2) + \ldots\ldots + 1$

$=$ Sum of first n natural numbers $= \frac{n(n+1)}{2}$

10. (d): Let the numbers be $a - 3d, a - d, a + d, a + 3d$.

Sum of four terms $= 48 \Rightarrow a = 12$

Also, $\frac{(12-3d)(12+3d)}{(12-d)(12+d)} = \frac{27}{35}$

$\Rightarrow$ $35(16 - d^2) = 3(144 - d^2) \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

$\therefore$ The numbers are $12 - 6, 12 - 2, 12 + 2, 12 + 6$ or $12 + 6, 12 + 2, 12 - 2, 12 - 6 = 6, 10, 14, 18$ or $18, 14, 10, 6$

$\therefore$ Largest term is 18.

11. (d): Let $f(x) = \frac{1 - \cos x\sqrt{\cos 2x}}{x^2}$

$$= \frac{1 - \cos x\sqrt{\cos 2x}}{x^2} \times \frac{1 + \cos x\sqrt{\cos 2x}}{1 + \cos x\sqrt{\cos 2x}}$$

$$= \frac{1 - \cos^2 x \cos 2x}{x^2} \times \frac{1}{1 + \cos x\sqrt{\cos 2x}}$$

$$\therefore \quad \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(1 - \cos^2 x(1 - 2\sin^2 x))}{x^2} \times \lim_{x\to 0} \frac{1}{1 + \cos x\sqrt{\cos 2x}}$$

$$= \lim_{x\to 0}\left(\frac{1 - \cos^2 x + 2\sin^2 x \cos^2 x}{x^2}\right) \times \frac{1}{2}$$

$$= \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin^2 x(1 + 2\cos^2 x)}{x^2}\right)$$

$$= \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)\lim_{x\to 0}(1 + 2\cos^2 x)$$

$$= \frac{1}{2} \times (1+2) = \frac{3}{2} = \frac{k_1}{k_2} \quad \therefore \quad k_1^2 - k_2^2 = 5$$

12. (b): Sequence using the last element of the each group is $1^3, 3^3, 6^3, 10^3, \ldots$

Clearly t_n of 1, 3, 6, 10, ... is $\frac{1}{2} n(n+1)$

Now, if we replace n by $n - 1$, we get the last element of $(n-1)^{\text{th}}$ group

$\therefore$ Last element of the $(n-1)^{\text{th}}$ group $= \frac{1}{2} n(n-1)$, if we add 1 in it and cubing the number that number be the first element of n^{th} group. So the terms in the n^{th} group are

$$\left(\frac{n(n-1)}{2} + 1\right)^3, \left(\frac{n(n-1)}{2} + 2\right)^3, \ldots, \left(\frac{n(n+1)}{2}\right)^3$$

$\therefore$ Sum of the elements of n^{th} group

$=$ sum of the cubes of first $\frac{n(n+1)}{2}$ natural numbers

$-$ sum of the cubes of first $\frac{n(n-1)}{2}$ natural numbers

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2\left(\frac{n(n+1)}{2} + 1\right)^2}{4} - \frac{\left(\frac{n(n-1)}{2}\right)^2\left(\frac{n(n-1)}{2} + 1\right)^2}{4}$$

$$= \frac{n^3}{8}(n^2+1)(n^2+3) \qquad \text{(On simplification)}$$

13. (c) : $\because 2^{2017} + \alpha$

$= 2 \cdot 2^{2016} + \alpha = 2 \cdot 4^{(2\times 504)} + \alpha = 2 \cdot 16^{504} + \alpha$

$= 2(17 - 1)^{504} + \alpha$

$= 2[17\lambda + 1] + \alpha = 2 + \alpha +$ multiple of 17

For minimum value of α, we have $2 + \alpha = 17$

$\therefore \quad \alpha = 15$

14. (a) : We have, $\sum_{r=1}^{n} r^2 - \sum_{m=1}^{n} \sum_{r=1}^{m} r$

$$= \sum_{r=1}^{n} r^2 - \sum_{m=1}^{n} \frac{m(m+1)}{2} = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} \frac{r^2+r}{2}$$

$$= \frac{1}{2}\left[\sum_{r=1}^{n} (2r^2 - r^2 - r)\right] = \frac{1}{2}\left(\sum_{r=1}^{n} (r^2 - r)\right)$$

$$= \frac{1}{2}\left[(1^2 - 1) + (2^2 - 2) + (3^2 - 3) + \ldots. + (n^2 - n)\right]$$

$$= \frac{1}{2}\left[(1^2 + 2^2 + \ldots + n^2) - (1 + 2 + 3 + \ldots. + n)\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right] = \frac{1}{2}\frac{n(n+1)}{2}\left[\frac{2n+1}{3} - 1\right]$$

$$= \frac{1}{2}\frac{n(n+1)}{2}\frac{(2n-2)}{3} = \frac{(n+1)\cdot n \cdot (n-1)}{3 \cdot 2 \cdot 1}$$

$$= {}^{n+1}C_3$$

15. (a) : Let $P(r) = (r + 2)(r + 3)(r + 4)(r + 5)\ \forall\ r \in N$

As $r \in N$, $P(r)$ is the product of four consecutive positive integers, which means $P(r)$ is the multiple of 4!.

$\therefore$ Greatest positive integer which divides $(r + 1)(r + 2)(r + 3)(r + 4)$ is 4!

16. (d): t_{r+1} term in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

$$= \binom{11}{r}\left(\frac{1}{bx}\right)^r (ax^2)^{11-r} = \binom{11}{r} a^{11-r} \cdot b^{-r} \cdot x^{22-3r}$$

Now, $22 - 3r = 7 \Rightarrow r = 5$.

$\therefore$ The coefficient is $\binom{11}{6} a^6 b^{-5}$ ...(i)

Also, t_{r+1} term in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$

$$= \binom{11}{r}(ax)^{11-r}\left(-\frac{1}{bx^2}\right)^r = \binom{11}{r} a^{11-r}(-b)^{-r} x^{11-3r}$$

Now, $11 - 3r = -7 \Rightarrow r = 6$

$\therefore$ The coefficient is $\binom{11}{6} a^5 b^{-6}$ ...(ii)

Since, the coefficients are equal, from (i) and (ii), we get $a = b^{-1} \Rightarrow ab = 1$

17. (c) : $m(16)^{15} = (16)^{15} + 2 \cdot 17(16)^{14} + 3(17)^2(16)^{13} + \ldots.. + 16^1(17)^{15}$

$$\therefore \quad m = 1 + \frac{2 \cdot 17(16)^{14}}{(16)^{15}} + 3 \cdot \frac{(17)^2(16)^{13}}{(16)^{15}} + \ldots\ldots + \frac{16(17)^{15}}{(16)^{15}}$$

$$\text{or} \quad m = 1 + 2\left(\frac{17}{16}\right) + 3\left(\frac{17}{16}\right)^2 + \ldots.. + 16\left(\frac{17}{16}\right)^{15}$$

$$\text{or} \quad m = 1 + 2x + 3x^2 + \ldots. + 16x^{15} \quad \left(\text{where } x = \frac{17}{16}\right)$$

$$= \frac{d}{dx}(x^1 + x^2 + \ldots\ldots. + x^{16})$$

$$= \frac{d}{dx}\left[\frac{x(x^{16} - 1)}{x - 1}\right]$$

$$= \frac{(x-1)(17x^{16} - 1) - x(x^{16} - 1)}{(x-1)^2} = \frac{16x^{17} - 17x^{16} + 1}{(x-1)^2}$$

$$\Rightarrow \quad m = \frac{1}{(x-1)^2} \quad \left(\because 16x^{17} = 17x^{16} \text{ where } x = \frac{17}{16}\right)$$

$\Rightarrow \quad m = 256$

$\therefore \quad m + 33 = 289 \Rightarrow \sqrt{m + 33} = 17$

18. (d): $\lim_{n\to\infty} \frac{1^m + 2^m + 3^m + \ldots + n^m}{n^{m+1}}$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^m}{n^{m+1}}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{r}{n}\right)^m = \int_0^1 x^m dx = \frac{1}{m+1}$$

19. (c) : Let $f(x) = a + b(x - 1) + c(x - 1)^2 + d(x - 1)^3$

$f(1) = 1 \Rightarrow a = 1,\ f'(1) = 2 \Rightarrow b = 2$

$f''(1) = 3 \Rightarrow c = \frac{3}{2}, f'''(1) = 4 \Rightarrow d = \frac{2}{3}$

$\therefore f(x) = 1 + 2(x - 1) + \frac{3}{2}(x - 1)^2 + \frac{2}{3}(x - 1)^3$

$f(0) = 1 - 2 + \frac{3}{2} - \frac{2}{3} = -1 + \frac{5}{6} = -\frac{1}{6}$

20. (a) : $y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)} = \frac{y - x}{x + y}$

$$\therefore \quad y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{y - x}{x + y}\right)$$

$$=\frac{(x+y)(y'-1)-(y-x)(1+y')}{(x+y)^2}$$

$\Rightarrow\ (x+y)^2\,y''=2(xy'-y)\Rightarrow\lambda=2$

21. (325) : Given, $\sum_{i=1}^{25} a_i=a_1+a_2+a_3+\ldots+a_{25}$

$=\frac{25}{2}(a_1+a_{25})\ \Rightarrow\ 625=\frac{25}{2}(a_1+a_{25})$

$\therefore\ \ a_1+a_{25}=50\Rightarrow 2a_{13}=50\Rightarrow a_{13}=25$

$\therefore\ \ \sum_{j=1}^{13} a_{2j-1}=a_1+a_3+a_5+\ldots+a_{25}$

$=6(a_1+a_{25})+a_{13}=6\times 50+25=325$

22. (3) : The function $\log|x|$ is not defined at $x=0$. Again, for $f(x)$ to be defined, $\log|x|\neq 0\ \therefore\ x\neq\pm 1$

$\therefore$ Number of points of discontinuity are $0, -1, 1$ *i.e.* 3.

23. (8/6) : Number of terms in the expansion of $(x_1+x_2+\ldots+x_r)^n$ is given by ${}^{n+r-1}C_{r-1}$

$\therefore$ Required number of terms in the given expansion

$={}^{15+4-1}C_{4-1}$ $(\because\ n=15, r=4)$

$={}^{18}C_3=816$

24. (1) : $P(n)=3^{2n}=9^n=(1+8)^n$

$P(n)=1+8m$ (Using Binomial theorem)

$\therefore$ Remainder is 1.

Short Cut Method : $P(n)=3^{2n}$

Now, $2n$ = even and every even power of an odd integer, when divided by 8 always leaves the remainder one.

25. (97) : $\frac{3-5x}{(1-x)^2}=(3-5x)(1-x)^{-2}$

$=(3-5x)(1+2x+3x^2+\ldots+50x^{49}+51x^{50}+\ldots)$

$\therefore$ Required coefficient $=(3\times 51)-(5\times 50)$

$=153-250=-97$

26. (1) : $g=\sqrt{ab}$ and a, p, q, b are in A.P.

Common difference (d) is $\frac{b-a}{3}$.

$\therefore\ \ p=a+d=a+\frac{b-a}{3}=\frac{2a+b}{3}$,

$q=b-d=b-\frac{b-a}{3}=\frac{a+2b}{3}$

Now, $(2p-q)(p-2q)$

$=\frac{(4a+2b-a-2b)}{3}\cdot\frac{(2a+b-2a-4b)}{3}=-ab=-g^2$

$\therefore\ \ \left|\frac{(2p-q)(p-2q)}{g^2}\right|=|-1|=1$

27. (2) : $\underset{x\to 0}{\text{Lt}}\ \frac{Ae^x-B\cos x+Ce^{-x}}{x\sin x}=2$ (given)

$$\Rightarrow\ \underset{x\to 0}{\text{Lt}}\ \frac{A\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\right)-B\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)+C\left(1-x+\frac{x^2}{2!}-\ldots.\right)}{x^2\,\frac{\sin x}{x}}=2$$

$$\Rightarrow\ \underset{x\to 0}{\text{Lt}}\left(\frac{A-B+C}{x^2}+\frac{A-C}{x}+\frac{A+B+C}{2}+\ldots\right)\cdot\underset{x\to 0}{\text{Lt}}\ \frac{1}{\frac{\sin x}{x}}$$

$=2$

$\Rightarrow$ L.H.S will exist only if

$A-B+C=0$ or $A+C=B, A-C=0$ and $\frac{A+B+C}{2}=2$

$\therefore\ \ B=2$ as $A+C=B$

28. (4) : We have, $(32)^{32}=(2^5)^{32}=2^{160}=(3-1)^{160}$

$=3\lambda+1, \lambda\in N$

Again, $(32)^{(32)^{(32)}}=(32)^{3\lambda+1}=2^{15\lambda+5}$

$=2^{15\lambda+3+2}=4\cdot 2^{3(5\lambda+1)}$

$=4(8)^{5\lambda+1}=4[(7+1)^{5\lambda+1}]=4(7m)+4$

$\therefore$ Remainder = 4

29. (3) : Let S be the sum of the given series.

$\therefore\ \ S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots.$ to ∞

$\Rightarrow\ (S-1)=\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ to ∞ ...(i)

$\Rightarrow\ (S-1)\times\frac{1}{3}=\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+\ldots$ to ∞ ...(ii)

Subtracting (ii) from (i), we get

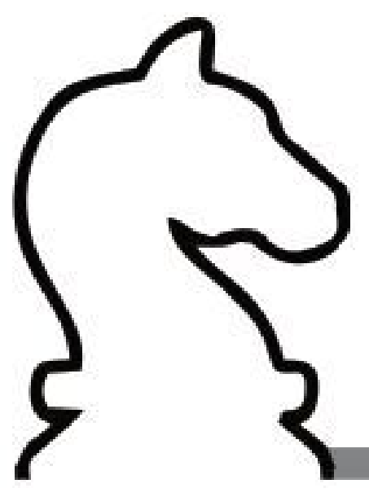
$\frac{2}{3}(S-1)=\frac{2}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\ldots$ to ∞

$\Rightarrow\ \frac{2}{3}(S-1)=\frac{2}{3}+\frac{\frac{4}{3^2}}{1-\frac{1}{3}}\ \Rightarrow\ \frac{2}{3}(S-1)=\frac{2}{3}+\frac{2}{3}\ \Rightarrow\ S=3$

30. (1) : $\underset{x\to 0}{\text{Lt}}\ \frac{\log(1+x^3)}{\sin^3 x}=\underset{x\to 0}{\text{Lt}}\ \frac{\log(1+x^3)}{x^3}\cdot\frac{x^3}{\sin^3 x}$

$=\underset{x\to 0}{\text{Lt}}\ \frac{\log(1+x^3)}{x^3}\ \underset{x\to 0}{\text{Lt}}\ \frac{x^3}{\sin^3 x}=1\times 1=1$

Challenging PROBLEMS ON JEE



SECTION-1 (SINGLE ANSWER TYPE)

1. Number of positive real solutions of equation $x^5 - 10x + 35 = 0$ is
(a) 0 (b) 1
(c) 2 (d) none of these

2. The least value of a such that $f(x) \geq 0 \ \forall \ x \in R$, where $f(x) = x^4 - 2x^3 + ax^2 - 2x + 1$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4

3. The maximum value of $f(x)$, where
$f(x) = 21\sqrt[4]{x} - 42^{\log_{16} x} + 2 \cdot 21^{\log_{16} x}, x > 0$ is
(a) 42 (b) 21 (c) 14 (d) 7

4. If $5x^2 - 2kx + 1 < 0$ has exactly one integral solution which is '1' then find the sum of all positive integral values of k.
(a) 3 (b) 9 (c) 6 (d) 12

5. The number of solutions, of the equation $(x - 2) + 2\log_2(2^x + 3x) = 2^x$ is
(a) 5 (b) 6 (c) 3 (d) 2

6. If the roots of $x^4 - ax^3 + bx^2 - cx + d$ are real and positive, then the minimum value of $\frac{(a+b+c)}{2\sqrt{d}}$ is
(a) 5 (b) 6 (c) 7 (d) 8

7. If the quadratic equation $mx^2 - nx + 12 = 0$, where m and n are positive integers not exceeding 10, has both roots greater than 2, then the number of possible ordered pairs (m, n) is
(a) 3 (b) 2 (c) 1 (d) 7

8. The number of integers k for which equation $x^3 - 27x + k = 0$ has atleast two distinct integral roots is
(a) 1 (b) 2 (c) 3 (d) 4

9. Let z be a complex number such that $\left|3z + \frac{1}{z}\right| = 1$ and $\arg(z) = \phi$, then the maximum value of $\cos^2\phi$ equals
(a) 1/2 (b) 2/3 (c) 1/6 (d) 1/12

10. The minimum value of $a^2 + b^2 - \sqrt{ab}$ as a, b take positive real values is m. Let $a^2 + b^2 - \sqrt{ab} = m$. Then 4 is the integral multiple of
(a) 2 (b) $\frac{1}{a} + \frac{1}{b}$
(c) $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$ (d) $a - b$

11. The mirror image of the curve given by $\operatorname{Arg}\left(\frac{z+i}{z-1}\right) = \frac{-\pi}{4}$ in the line $(1+i)\bar{z} + (i-1)z = 0$ is
(a) $\operatorname{Arg}\left(\frac{z+1}{z-i}\right) = -\frac{\pi}{4}$ (b) $\operatorname{Arg}\left(\frac{z+1}{z-i}\right) = \frac{\pi}{4}$
(c) $\operatorname{Arg}\left(\frac{z+1}{iz+1}\right) = \frac{\pi}{4}$ (d) none of these

12. If $x_1^2 + y_1^2 = x_2^2 + y_2^2 = x_3^2 + y_3^2 = 4$, where $x_i y_i \in R$, $i = 1, 2, 3$ then the maximum value of
$(x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2 + (y_1 - y_2)^2 + (y_2 - y_3)^2 + (y_3 - y_1)^2$ equals
(a) 18 (b) 24 (c) 36 (d) 48

13. If $z_1, z_2, z_3 \in C$ satisfy the system of equation given by $|z_1| = |z_2| = |z_3| = 1$, $z_1 + z_2 + z_3 = 1$ and $z_1 \cdot z_2 \cdot z_3 = 1$ such that $\operatorname{Im}(z_1) < \operatorname{Im}(z_2) < \operatorname{Im}(z_3)$, then find the value of $[|z_1 + z_2^2 + z_3^3|]$.
(where $[\cdot]$ denotes the greatest integer function)
(a) 1 (b) 2 (c) 3 (d) 4

SECTION-2 (INTEGER ANSWER TYPE)

14. Let $P(x) = x^2 + bx + c$, where b and c are integers. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then find the value of $P(1)$.

15. The set of real parameter 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real solutions, is given by $\left[\frac{m}{n}, \infty\right)$, where m and n are relatively prime positive integers, then the value of $(m + n)$ is equal to ______.

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16. If x is real, then the maximum value of $y=2(3-x)(x+\sqrt{x^2+4})$ is k, then $(k-10)$ =_____.

17. $a_i, b_i, c_i \in R^+ \ \forall\ i \in N$. $a_1 = b_1 = 1$ and $b_n = a_n b_{n-1} - 2$ for $n > 1$. If $\lim\limits_{n\to\infty} b_n = 5$ and $c_i = \frac{1}{a_i}$, then twice of $(c_1 + c_1c_2 + c_1c_2c_3 + \ldots.. \infty)$ is equal to _____

18. Let p, q be real numbers such that $p^2 + q > q^2 + p$. Then least value of $[8p^2 + 8q] + 10$ is (where $[\cdot]$ denotes the greatest integer function) _____.

19. If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal real roots, then the value of $6a^5 + c^5$, is L and the value of $\frac{b^4}{a^4}$, is M,then the value of $\frac{L+M}{3}$ is_____

20. Given the cubic equation $x^3 - 2kx^2 - 4kx + k^2 = 0$. If one root of the equation is less than 1, other root is in the interval (1, 4) and the 3rd root is greater than 4, then the value of k lies in the interval $(a+\sqrt{b}, b(a+\sqrt{b}))$, where $a, b \in N$. Find the value of $(a+b)^3 + (ab+2)^2 - 2000$

21. If a complex number α satisfies the equation $(39z-12\overline{z})^4 = -1$, then the value of $\left[\frac{|Re(\alpha)|^{-1}+|\mathrm{Im}(\alpha)|^{-1}}{13}\right]$ is (where $[\cdot]$ denotes greatest integer function) _____.

22. If α, β and γ are three distinct real values such that

$$\frac{\sin\alpha+\sin\beta+\sin\gamma}{\sin(\alpha+\beta+\gamma)}=\frac{\cos\alpha+\cos\beta+\cos\gamma}{\cos(\alpha+\beta+\gamma)}=2$$

and $\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) = a$, then find the value of $\lim\limits_{x\to a}\frac{\sqrt{x^2-a^2}}{\sqrt{x-a}+(\sqrt{x}-\sqrt{a})}$.

23. For the equation $z^6 - z^3 - 2450 = 0$, where z is a complex number. The number of roots having positive real part is m, negative real part is n, positive imaginary part is p, negative imaginary part is q, then the value $(m + n) - (p + q)$ equals to_____.

24. Area of the region represented by the intersection of $\overline{z}(z-2-3i)-z(2-3i)+12 \ge 0$ and $|z+\overline{z}-4| \le 2$ and $|z-\overline{z}-6i| \le 2$, in argand plane is M sq. units, then $[2M]$ = _____. ($[\cdot]$ denotes Greatest integer function)

SOLUTIONS

1. (a) : $f(x) = x^5 - 10x + 35$
$f(0) = 35, f'(x) = 5x^4 - 10 \ge -10$
$\Rightarrow\ y' + 10 \ge 0$
$\Rightarrow\ (y + 10x) \ge (y + 10x)_{\text{at } x=0}$
$\Rightarrow\ y + 10x \ge 35 \ \Rightarrow\ f(x) \ge -10x + 35\ \forall\ x \in R$
$\Rightarrow\ f(x) \ge 0\ \forall\ x \in (0, 3)$
Also, $f(3) > 0$ and $f'(x) > 0\ \forall\ x > 3$
Hence, $f(x) > 0\ \forall\ x \in R^+$

2. (b) : $f(x) = (x^2 - x)^2 + (x - 1)^2 + (a - 2)x^2$
$= (x - 1)^2 (x^2 + 1) + (a - 2)x^2 \ge 0$, which is possible if $a \ge 2$.

3. (a) : $21\cdot16^{\log_{16}x^{1/4}} - 2^{\log_{16}x}21^{\log_{16}x} + 2\cdot21^{\log_{16}x}$

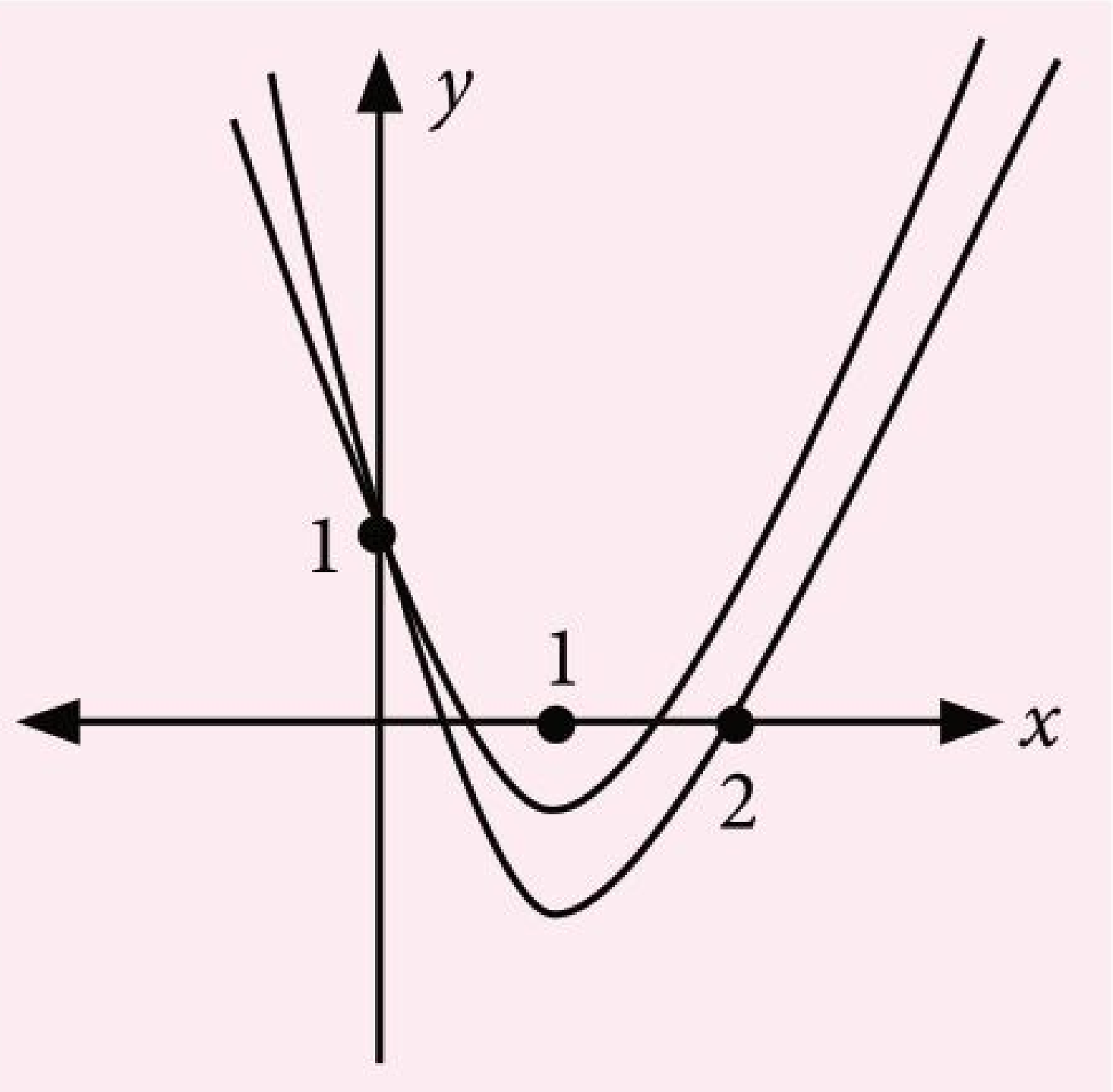
$$= 21\cdot2^{\log_{16}x} - 2^{\log_{16}x}\cdot21^{\log_{16}x} + 2\cdot21^{\log_{16}x}$$

$$= 2^{\log_{16}x}(21-21^{\log_{16}x}) + 2(21^{\log_{16}x}-21) + 42$$

$$= (21 - 21^{\log_{16}x})(2^{\log_{16}x} + 2) + 42$$

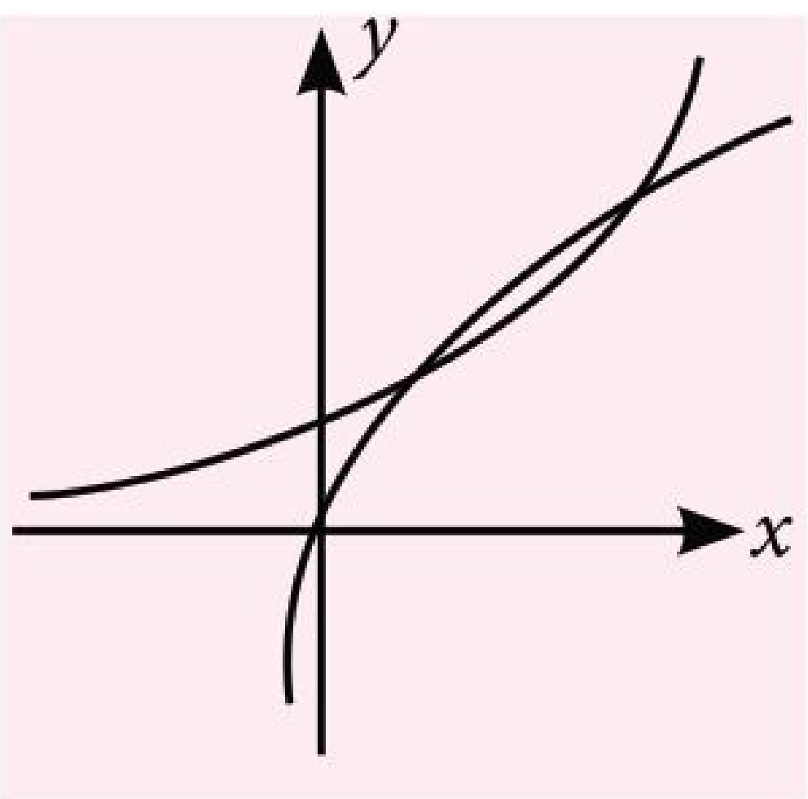
$\therefore$ Maximum value is 42.

4. (b) :



$f(1) < 0$
$\Rightarrow\ 5 - 2k + 1 < 0 \ \Rightarrow\ 6 < 2k \ \Rightarrow\ k > 3$
Also, $f(2) \ge 0$
$\Rightarrow\ 20 - 4k + 1 \ge 0$
$\Rightarrow\ 21 \ge 4k \ \Rightarrow\ \frac{21}{4} \ge k$
$\therefore$ Values of k = 4, 5.

5. (d) : Let $2^x > 3x$
$\Rightarrow\ 2^{x+1} > 2^x + 3x$
$\therefore\ (x - 2) + 2\log_2(2^x + 3x) < (x - 2) + 2\log_2(2^{x+1})$
$= x - 2 + 2x + 2 = 3x$
$\therefore\ 2^x < 3x$, which is contradiction
$\therefore\ 2^x = 3x$
$\therefore$ Two solutions exist.



6. (c) : We have, $a = \alpha + \beta + \gamma + \delta$; $b = \Sigma\alpha\beta$, $c = \Sigma\alpha\beta\gamma$ and $d = \alpha\beta\gamma\delta$

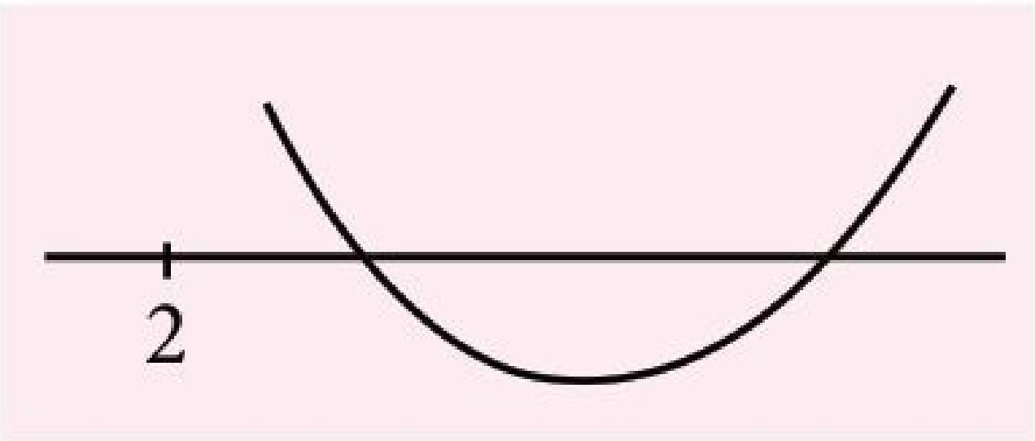
$$\frac{a+b+c}{14}=\frac{\alpha+\beta+\gamma+\delta+\Sigma\alpha\beta+\Sigma\alpha\beta\gamma}{14}\ge\sqrt[14]{\alpha^7\beta^7\gamma^7\delta^7}=\sqrt{d}$$

7. (c) : Since, m, n are positive integer which are less than or equal to 10, then $1 \le m, n \le 10$
If the roots of equation $ax^2 + bx + c = 0$ are greater than 2, the following conditions satisfied.

(i) $D \ge 0$

(ii) $-\frac{b}{2a} > 2$

(iii) $f(2) > 0$



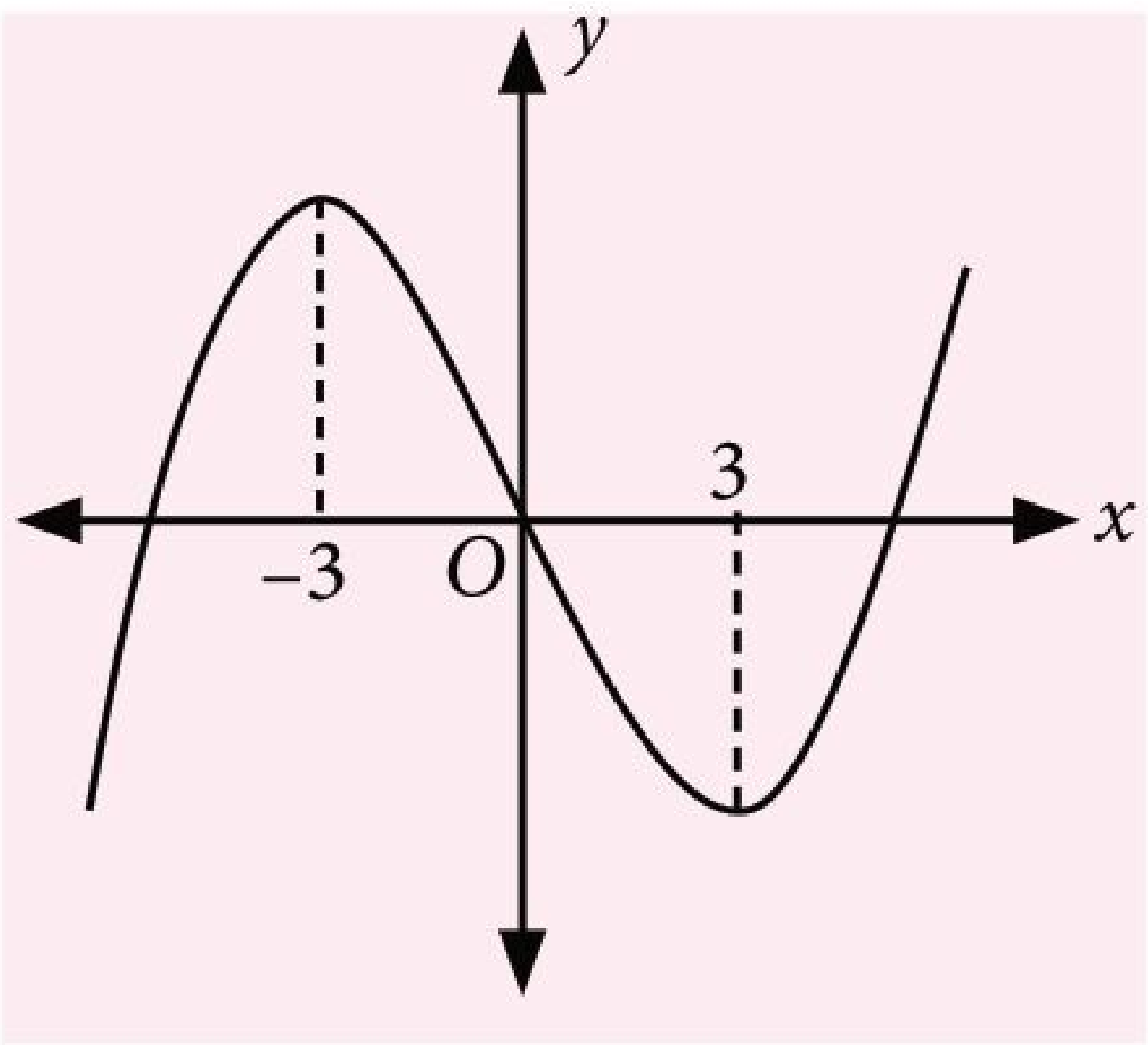
$\therefore \quad D \ge 0 \Rightarrow n^2 - 48m \ge 0$ ...(1)

$-\frac{b}{2a} > 2 \Rightarrow n > 4m$ ...(2)

$f(2) > 0 \Rightarrow 2m - n + 6 > 0$ ...(3)

$\therefore$ From (1) $\cap$ (2) $\cap$ (3), we get (1, 7) is the only possible ordered pair.

8. (b) : $f(x) = x^3 - 27x + k = 0$
$f'(x) = 3x^2 - 27 = 0$
$\Rightarrow \quad x = \pm 3$
and $f''(x) = 6x = 0$
$\Rightarrow \quad x = 0$
Since, 2 roots are integer
$\Rightarrow$ 3rd root must be integer



Possible cases are :
$x = 0$
$\Rightarrow \quad k = 0$ (not possible);
$x = \pm 1 \Rightarrow k = \pm 26$ (not possible)
$[\because \quad x^3 - 27x + 26 = 0 \Rightarrow (x - 1)(x^2 + x - 26) = 0]$
$x = \pm 2 \Rightarrow k = \pm 46$ (not possible)
$x = \pm 3 \Rightarrow k = \pm 54$
At $k = 54$, $x^3 - 27x + 54 = 0 \Rightarrow (x - 3)^2(x + 6) = 0$
At $k = -54$, $x^3 - 27x - 54 = (x + 3)^2(x - 6)$
$\therefore$ Only possibility is $k = \pm 54$.

9. (d) : Let $z = r\cos\phi + r\sin\phi$

$$\therefore \quad \frac{1}{z} = \frac{\cos\phi - i\sin\phi}{r}$$

We have, $\left|3z + \frac{1}{z}\right| = 1$

$$\Rightarrow \left|\left(3r + \frac{1}{r}\right)\cos\phi + i\left(3r - \frac{1}{r}\right)\sin\phi\right| = 1$$

$$\Rightarrow \left(3r + \frac{1}{r}\right)^2 \cos^2\phi + \left(3r - \frac{1}{r}\right)^2 \sin^2\phi = 1$$

$$\Rightarrow \left(3r + \frac{1}{r}\right)^2 \cos^2\phi + \left(3r - \frac{1}{r}\right)^2 (1 - \cos^2\phi) = 1$$

$$\Rightarrow \cos^2\phi\left\{\left(3r + \frac{1}{r}\right)^2 - \left(3r - \frac{1}{r}\right)^2\right\} = 1 - \left(3r - \frac{1}{r}\right)^2$$

$$\Rightarrow 12\cos^2\phi \le 1 \Rightarrow \cos^2\phi \le \frac{1}{12}$$

10. (c) : We have, $a^2 + b^2 - \sqrt{ab}$

$$= a^2 + b^2 - 2ab + 2ab - \sqrt{ab}$$

$$= (a - b)^2 + 2\left(ab - \frac{\sqrt{ab}}{2}\right)$$

$$= (a - b)^2 + 2\left(ab - \frac{\sqrt{ab}}{2} + \frac{1}{16}\right) - \frac{1}{8}$$

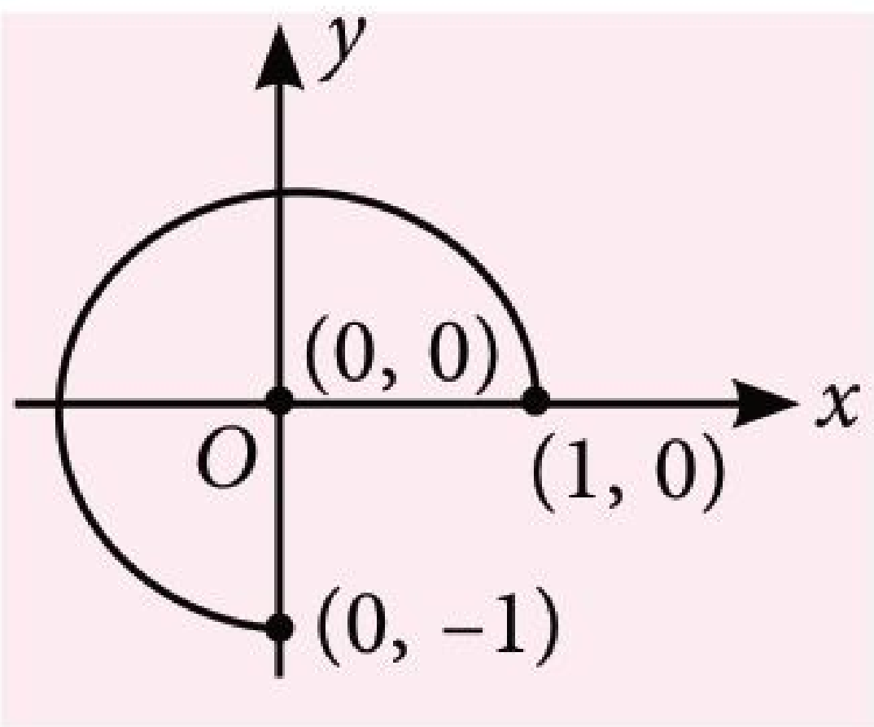
$$= (a - b)^2 + 2\left(\sqrt{ab} - \frac{1}{4}\right)^2 - \frac{1}{8}$$

Hence, $m = -\frac{1}{8}$ and $a = b$, $\sqrt{ab} = \frac{1}{4}$ at minimum value

Hence $a = b = \frac{1}{4}$

11. (b) : Clearly, z lies on $|z| = 1$
$\Rightarrow \quad z\overline{z} = 1$
Given line is $x = y$
The image of z in $x = y$ is

$i\overline{z}$ *i.e.*, $\frac{i}{z}$



$\therefore$ Required curve is

$$\arg\left(\frac{\frac{i}{z} + i}{\frac{i}{z} - 1}\right) = -\frac{\pi}{4} \Rightarrow \arg\left(\frac{i(1 + z)}{i - z}\right) = -\frac{\pi}{4}$$

$$\Rightarrow \arg(-i) + \arg\left(\frac{z + 1}{z - i}\right) = -\frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{z + 1}{z - i}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

12. (c) : Given expression can be written as
$a^2 + b^2 + c^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$
$= 4(2)^2 (\sin^2 A + \sin^2 B + \sin^2 C)$

$$= 16\sum\sin^2 A \le 16\left(\frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right) \le 36$$

13. (b) : Consider, $z_1z_2 + z_2z_3 + z_3z_1$

$$= z_1z_2z_3\left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right)$$

$$= 1\left(\frac{|z_1|^2}{z_1} + \frac{|z_2|^2}{z_2} + \frac{|z_3|^2}{z_3}\right) = \overline{z}_1 + \overline{z}_2 + \overline{z}_3$$

$$= \overline{(z_1 + z_2 + z_3)} = 1$$

$\therefore$ The cubic equation with roots z_1, z_2 and z_3 in z is given by

$(z - z_1)(z - z_2)(z - z_3) = 0$

$\Rightarrow$ $z^3 - (\Sigma z_1) z^2 + (\Sigma z_1 z_2)z - z_1 z_2 z_3 = 0$

$\Rightarrow$ $z^3 - 1z^2 + 1z - 1 = 0$

$\Rightarrow$ $(z - 1)(z^2 + 1) = 0$

$\Rightarrow$ $z = 1, \pm i$

$\because$ $\text{Im}(z_1) < \text{Im}(z_2) < \text{Im}(z_3)$

$\therefore$ $z_1 = -i, z_2 = 1, z_3 = i$

$$\therefore \left|z_1 + z_2^2 + z_3^3\right| = |-i + 1^2 + i^3| = |1 - 2i| = \sqrt{5}$$

$$\therefore \left[\left|z_1 + z_2^2 + z_3^3\right|\right] = [2.24] = 2$$

14. (4) : Since, $P(x)$ divides both equation $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$.

Hence $P(x)$ also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$,

which is a quadratic.

Hence $P(x) = x^2 - 2x + 5$

$\therefore$ $P(1) = 4$

15. (7) : We have, $a^2 - (2x^2 + 1)a + x^4 + x = 0$

$$\therefore \quad a = \frac{(2x^2+1) \pm \sqrt{(2x^2+1)^2 - 4(x^4+x)}}{2}$$

$$\Rightarrow \quad a = \frac{(2x^2+1) \pm \sqrt{4x^2 - 4x + 1}}{2}$$

$\Rightarrow$ $2a = (2x^2 + 1) \pm (2x - 1)$

$\Rightarrow$ $a = x^2 + x, x^2 - x + 1$

For $x^2 + x - a = 0$

$D \geq 0 \Rightarrow 1 + 4a \geq 0$ [$\because$ Roots are real]

$$\Rightarrow \quad a \geq -\frac{1}{4}$$

For $x^2 - x + 1 - a = 0$

$$D \geq 0 \Rightarrow -3 + 4a \geq 0 \Rightarrow a \geq \frac{3}{4}$$

$\therefore$ Required set is $\left[\frac{3}{4}, \infty\right)$

$\therefore$ $m + n = 7$

16. (3) : Consider, $2(a - x)(x + \sqrt{x^2 + b^2})$

Put $\sqrt{x^2 + b^2} + x = t$ ...(1)

$$\Rightarrow \quad \frac{b^2}{\sqrt{x^2+b^2} - x} = t$$

$$\Rightarrow \quad \sqrt{x^2+b^2} - x = \frac{b^2}{t} \quad \text{...(2)}$$

Subtracting (ii) from (i), we get $2x = t - \dfrac{b^2}{t}$

From (*), we get $\left(2a - \left(t - \dfrac{b^2}{t}\right)\right)t$

$= 2at - t^2 + b^2 = -a^2 + 2at - t^2 + b^2 + a^2$

$= a^2 + b^2 - (a - t)^2 \leq a^2 + b^2$

17. (3) : Let $S = 1 + \dfrac{b_1}{(b_2+2)} + \dfrac{b_1 b_2}{(b_2+2)(b_3+2)}.....$

as $a_n = \dfrac{b_n + 2}{b_{n-1}}$

Multiply by 2 and divide by $(b_1 + 2)$ which is 3.

Then write 2 as $2 + b_i - b_i$ for each term

We get $2S_n = 3\left(1 - \prod_{i=1}^{n} \dfrac{b_i}{b_i + 2}\right)$ also let $b_i < M \ \forall \ i$,

$$0 < \frac{b_i}{b_i+2} < \frac{M}{M+2}$$

SAMURAI SUDOKU

SOLUTION - OCTOBER 2021

2	6	1	3	4	9	8	7	5				7	2	6	4	3	8	5	9	1
4	3	7	8	1	5	2	6	9				9	3	1	5	2	7	6	4	8
8	5	9	2	7	6	3	1	4				8	5	4	6	9	1	7	2	3
6	4	2	7	8	3	9	5	1				3	9	7	8	5	2	4	1	6
3	1	5	6	9	2	7	4	8				1	4	8	9	7	6	3	5	2
7	9	8	4	5	1	6	3	2				2	6	5	3	1	4	9	8	7
5	2	3	1	6	8	4	9	7	3	5	8	6	1	2	7	4	5	8	3	9
1	8	4	9	3	7	5	2	6	9	7	1	4	8	3	2	6	9	1	7	5
9	7	6	5	2	4	1	8	3	4	2	6	5	7	9	1	8	3	2	6	4
						8	3	4	6	9	2	1	5	7						
						6	7	9	1	3	5	8	2	4						
						2	5	1	8	4	7	3	9	6						
4	5	6	7	8	3	9	1	2	5	6	4	7	3	8	9	4	2	5	6	1
7	8	9	4	2	1	3	6	5	7	8	9	2	4	1	5	3	6	8	7	9
3	2	1	6	9	5	7	4	8	2	1	3	9	6	5	7	1	8	4	2	3
6	4	5	2	1	7	8	9	3				8	1	3	2	6	5	9	4	7
1	9	3	5	6	8	2	7	4				6	7	4	1	8	9	2	3	5
8	7	2	9	3	4	1	5	6				5	2	9	4	7	3	1	8	6
9	6	7	8	5	2	4	3	1				1	9	7	3	2	4	6	5	8
5	1	8	3	4	9	6	2	7				3	8	2	6	5	1	7	9	4
2	3	4	1	7	6	5	8	9				4	5	6	8	9	7	3	1	2

Then $3 < 2S_n < 3\left(1-\left(\frac{M}{M+2}\right)^n\right)$

Since $\lim\limits_{n\to\infty}\left(\frac{M}{M+2}\right)^n = 0$ applying squeeze play theorem, we get

$\lim\limits_{n\to\infty} 2S_n = 3$ (Note: $\lim\limits_{n\to\infty} b_n = 5$ just implies b_n is bounded)

18. (8) : We have, $p^2 + q > q^2 + p$

$\Rightarrow\ p^2 + q + q^2 + p < 2(p^2 + q)$

$\Rightarrow\ 0 \le p^2 + p + \frac{1}{4} + q^2 + q + \frac{1}{4} < 2(p^2+q) + \frac{1}{2}$

$\Rightarrow\ p^2 + q > -\frac{1}{4}$

$\therefore\ [8(p^2+q)] + 10 > \left[8\left(-\frac{1}{4}\right)\right] + 10 = 8$

19. (5) : Let $f(x) = x^5 - 10a^3x^2 + b^4x + c^5$

$\therefore\ f'(x) = 5x^4 - 20a^3x + b^4$

$f''(x) = 20x^3 - 20a^3 = 0$

$\Rightarrow\ x = a$ is three times repeated root

$f'(a) = 5a^4 - 20a^4 + b^4 = 0$

$\Rightarrow -15a^4 + b^4 = 0 \Rightarrow \frac{b^4}{a^4} = 15 = M$

$\therefore\ f(a) = a^5 - 10a^5 + b^4a + c^5 = 0$

$\Rightarrow\ a^5 - 10a^5 + 15a^5 + c^5 = 0$

$\Rightarrow\ 6a^5 + c^5 = 0 = L$

$\therefore\ \frac{L+M}{3} = \frac{15+0}{3} = 5$

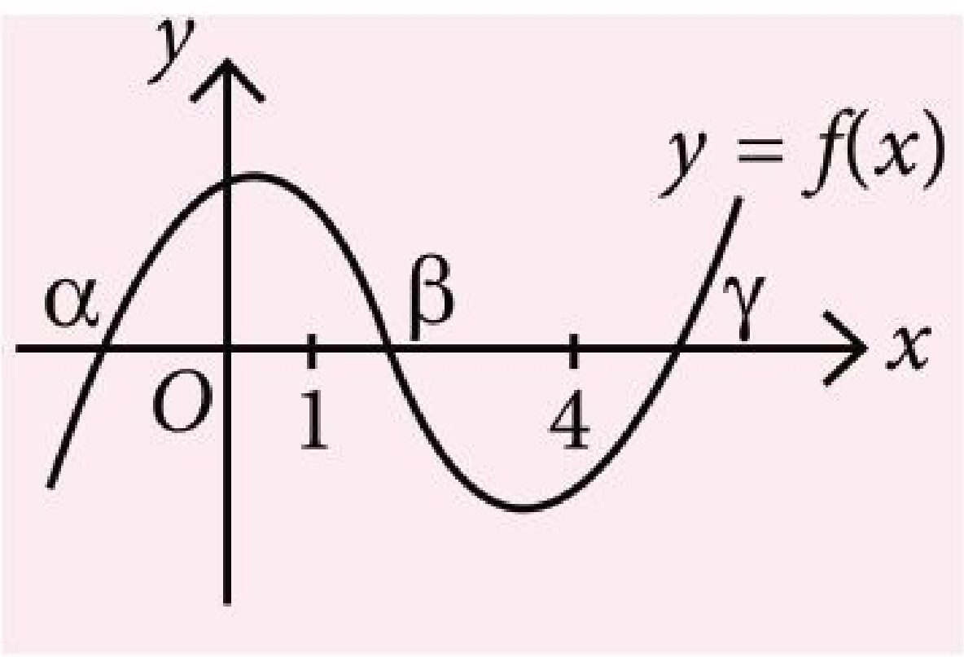
20. (7) : $f(x) = x^3 - 2kx^2 - 4kx + k^2 = 0$

note that $f(0) = k^2 > 0$

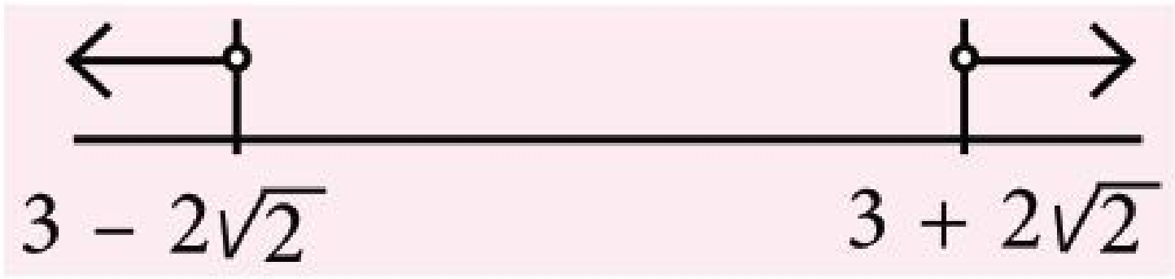
$f(1) > 0$

$\Rightarrow\ 1 - 2k - 4k + k^2 > 0$

$\Rightarrow\ k^2 - 6k + 1 > 0$



$\Rightarrow [k - 3 + 2\sqrt{2})][k - (3 - 2\sqrt{2})] > 0$



...(i)

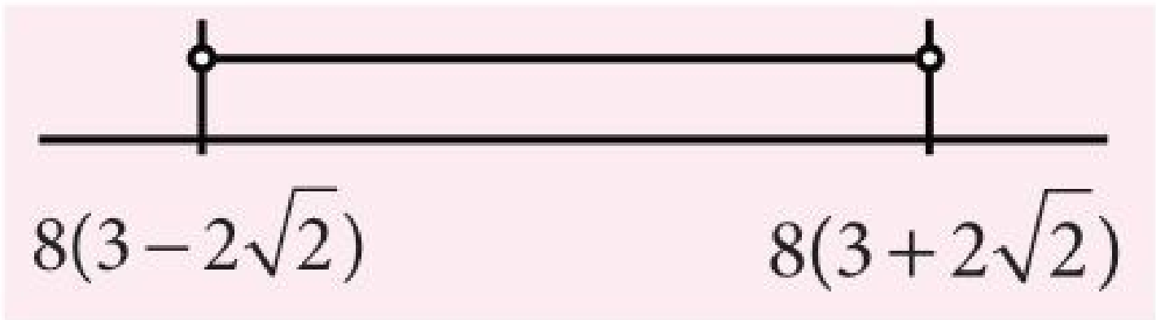
Also, $f(4) < 0$

$\Rightarrow\ 64 - 32k - 16k + k^2 < 0$

$\Rightarrow\ k^2 - 48k + 64 < 0 \Rightarrow (k-24)^2 < 512$

$\Rightarrow\ (k - 24 + 16\sqrt{2})(k - 24 - 16\sqrt{2}) < 0$

$\Rightarrow\ [k - 8(3-2\sqrt{2})]\ [k - 8(3+2\sqrt{2})] < 0$



...(ii)

From (i) and (ii), we get

$3 + 2\sqrt{2} < k < 8(3+2\sqrt{2})$

$\Rightarrow\ 3 + \sqrt{8} < k < 8(3+\sqrt{8})$

$\therefore\ a = 3$ and $b = 8$

Hence, $(a + b)^3 + (ab + 2)^2 - 2000$

$= (11)^3 + (26)^2 - 2000 = 1331 + 676 - 2000 = 7$

21. (8) : Let $z = a + ib \Rightarrow 27a + (51b)i = e^{i\frac{(2k+1)\pi}{4}}$

$\therefore\ a = \pm\frac{1}{27\sqrt{2}}, b = \pm\frac{1}{51\sqrt{2}}$

$\therefore\ \frac{|1\mathrm{Re}(\alpha)|^{-1} + |\mathrm{Im}(\alpha)|^{-1}}{13} = \left[\frac{27\sqrt{2}+51\sqrt{2}}{13}\right] = 8$

22. (2) : Put, $a = \text{cis}\,\alpha, b = \text{cis}\,\beta, c = \text{cis}\,\gamma$

$\Rightarrow\ a + b + c = 2abc \Rightarrow \sum\frac{1}{ab} = 2$

$\Rightarrow\ \Sigma\cos(\alpha+\beta) = 2$

$\because\ \frac{\sin\alpha + \sin\beta + \sin\gamma}{\sin(\alpha+\beta+\gamma)} = \frac{\cos\alpha + \cos\beta + \cos\gamma}{\cos(\alpha+\beta+\gamma)}$

$$= \frac{\sin(\alpha+\beta+\gamma)(\sin\alpha+\sin\beta+\sin\gamma) + \cos(\alpha+\beta+\gamma)(\cos\alpha+\cos\beta+\cos\gamma)}{\sin(\alpha+\beta+\gamma)\sin(\alpha+\beta+\gamma) + \cos(\alpha+\beta+\gamma)\cos(\alpha+\beta+\gamma)}$$

$\left(\because \frac{a}{b} = \frac{c}{d} = \frac{xa+yc}{xb+yd}\right)$ (By ratio and proportion)

$\therefore\ \frac{\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha)}{1} = 2$

$\Rightarrow\ a = 2$

23. (2) : $z^3 = 50, -49$

$\Rightarrow\ z^3 = -49 \Rightarrow z = \sqrt[3]{-49},\ \sqrt[3]{-49}\,\omega,\ \sqrt[3]{-49}\,\omega^2$

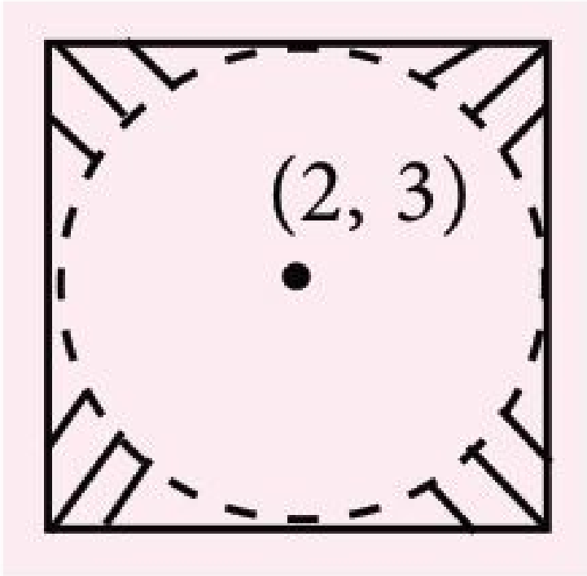
$\Rightarrow\ z^3 = 50 \Rightarrow z = \sqrt[3]{50},\ \sqrt[3]{50}\,\omega,\ \sqrt[3]{50}\,\omega^2$

$\therefore\ (m + n) - (p + q) = (3 + 3) - (2 + 2) = 2$

24. (1) : We have, $|z - (2 + 3\hat{i})| \ge 1$

and $2 \le |z + \bar{z}| \le 6$

and $4 \le |z - \bar{z}| \le 8$



Area of shaded region = 4 – π

$\therefore\ [8 - 2\pi] = 1$

◆

CONCEPT
MAP

STATISTICS

MATRICES

CONCEPT
MAP

Class XI Class XII

STATISTICS

Measures of Dispersion

Mean Deviation

It is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

$$\text{Mean Deviation} = \frac{\text{Sum of Deviations}}{\text{Number of Observations}}$$

For Ungrouped Data

Let $x_1, x_2, \ldots\ldots, x_n$ be n observations, then mean deviation about mean is given by $\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$;

Mean deviation about median is given by,

$$\text{M.D.}\,(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - M|$$

For Grouped Data

Let $x_1, x_2, \ldots\ldots, x_n$ be a set of n observations occurring with frequencies $f_1, f_2, \ldots, f_n$ respectively, then mean deviation about mean is given by $\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$;

Mean deviation about median is given by M.D.(M)

$$= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$$

Here, x_i are the mid-points of classes and $N = \sum_{i=1}^{n} f_i$ = Sum of frequencies

Shortcut Method

- **About mean :** $\text{M.D.}\,(\bar{x}) = \frac{1}{N}\cdot\sum_{i=1}^{n} f_i\,|x_i - \bar{x}|$, where mean, $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i\,d_i}{N} \times h$, a is assumed mean, $d_i = \frac{x_i - a}{h}$ and h = size of interval.
- **About median :** $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$, where median, $M = l + \frac{\frac{N}{2} - c.f.}{f} \times h$,

  N is sum of frequencies, l, f, h and $c.f.$ are respectively the lower limit, the frequency of the median class, the width of the class interval and the cumulative frequency of the class just preceding the median class.

Analysis of Frequency Distributions

If the given data has mean $(\bar{x})$ and standard deviation (σ), then

Coefficient of variation (C.V.) $= \frac{\sigma}{\bar{x}} \times 100, \bar{x} \neq 0$

The data whose C.V. is less is said to be more consistent.

Variance and Standard Deviation

Mean of the squares of the deviations from mean is called variance and is denoted by σ^2.

The positive square root of variance is known as standard deviation. It is denoted by σ.

For ungrouped data	$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
For grouped data	$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i\,(x_i - \bar{x})^2}$
Shortcut method	$\sigma = \frac{h}{N}\left[\sqrt{N\sum_{i=1}^{n} f_i\,u_i^2 - \left(\sum_{i=1}^{n} f_i\,u_i\right)^2}\right]$, where $u_i = \frac{x_i - a}{h}$, a = assumed mean, h = width of class-intervals

Properties of Standard Deviation

- S.D. is independent of change of origin.
- S.D. is not independent of change of scale.

Combined Variance of Two Series

If n_1, n_2 are the number of elements, $\bar{x}_1$, $\bar{x}_2$ are the means and σ_1, σ_2 are the standard deviations of two series respectively, then variance of combined series is

$$\sigma^2 = (\text{S.D.})^2 = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2},$$

where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$ and $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

MATRICES

A matrix is an ordered rectangular array of numbers or functions.
The numbers or functions are called the elements or entries of the matrix.

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m\times n}$, then A' or $A^T = [a_{ij}]_{n\times m}$

Properties

$(A')' = A$

$(AB)' = B'A'$

$(A \pm B)' = A' \pm B'$

$|A'| = |A|$

Operations on Matrices		Properties
Addition and Subtraction	$A \pm B = C$ *i.e.*, $[a_{ij}]_{m\times n} \pm [b_{ij}]_{m\times n}$ $= [a_{ij} \pm b_{ij}]_{m\times n}$ $= [c_{ij}]_{m\times n}$	• $A + B = B + A$ • $A + (B + C) = (A + B) + C$ • For any square matrix A, Additive inverse = - A • Additive identity = O
Scalar Multiplication	$k\,A = B$ *i.e.*, $[k\,a_{ij}] = [b_{ij}]$	• $k\,(A + B) = kA + kB$ • $(k + m)A = kA + mA$
Multiplication	$A_{m\times k} \times B_{k\times q} = C_{m\times q}$ *i.e.*, $\left[\sum_{r=1}^{k} a_{ir}\,b_{rj}\right] = [c_{ij}]$	• If AB exists, then BA may or may not exist. • AB may or may not be equal to BA. • $(AB)\,C = A(BC)$ • $I_m \times A_{m\times n} = A_{m\times n} = A_{m\times n} \times I_n$ • $A\,(B + C) = AB + AC$; $(B + C)\,A = BA + CA$

Types of Matrices

Column Matrix : $A = [a_{ij}]_{m\times 1}$

Row Matrix : $A = [a_{ij}]_{1\times n}$

Square Matrix : $A = [a_{ij}]_{m\times m}$

Diagonal Matix : $A = [a_{ij}]_{m\times m}$, where $a_{ij} = 0, \forall\, i \neq j$

Scalar Matrix : $A = [a_{ij}]_{n\times n}$, where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k

Zero Matrix : $A = [a_{ij}]$, where $a_{ij} = 0\ \forall\ i = j$ and $i \neq j$

Identity Matrix : $A = [a_{ij}]$, where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

Special Matrices

Nilpotent Matrix : $A^k = O$ and $A^{k-1} \neq O, k \in Z^+$
Here, k is order of nilpotent matrix.

Involutory Matrix : $A^2 = I$

Orthogonal Matrix : $AA^T = A^T A = I$, where A^T is the transpose of A.

Idempotent Matrix : $A^2 = A$

Periodic Matrix : $A^{k+1} = A, k \in Z$

Unitary Matrix : $AA^\theta = A^\theta A = I$, where A^θ denotes the conjugate transpose of A.

Symmetric Matrix : $A^T = A$

Skew Symmetric Matrix : $A^T = -A$
[where A is square matrix]

Inverse of a Matrix

If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.

Properties

- A^{-1} is unique for any square matrix A.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is

(a) $3x - 2y = 0$ (b) $x - 2y + 8 = 0$
(c) $2x - y - 2 = 0$ (d) $2x - 3y + 10 = 0$

2. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then the ratio of length of the sides of this triangle is

(a) $4:5:6$ (b) $5:9:13$
(c) $3:4:5$ (d) $5:6:7$

3. Number of roots of the equation $z^{10} - z^5 - 992 = 0$ with negative real part is

(a) 3 (b) 4
(c) 5 (d) an even number

4. Equation of line through a and ib, $a, b \in R$, $a, b \neq 0$ in complex form is

(a) $z\left(\frac{1}{2a} - \frac{i}{2b}\right) + \bar{z}\left(\frac{1}{2a} + \frac{i}{2b}\right) = 1$

(b) $z\left(\frac{1}{2a} + \frac{i}{2b}\right) + \bar{z}\left(\frac{1}{2a} + \frac{i}{2b}\right) = 1$

(c) $z\left(\frac{1}{2a} - \frac{i}{2b}\right) + \bar{z}\left(\frac{1}{2a} - \frac{i}{2b}\right) = 1$

(d) $z\left(\frac{1}{2a} + \frac{i}{2b}\right) + \bar{z}\left(\frac{1}{2a} - \frac{i}{2b}\right) = 1$

5. If α is the fifth root of unity, then

(a) $|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4| = 0$

(b) $|1 + \alpha + \alpha^2 + \alpha^3| = 1$

(c) $|1 + \alpha + \alpha^2| = 2\cos\frac{\pi}{5}$

(d) $|1 + \alpha| = 2\cos\frac{\pi}{10}$

6. The value of $\int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$ is

(a) $\frac{\pi}{2}\ln 2$ (b) ln2
(c) π ln2 (d) $\frac{\pi}{8}\ln 2$

7. The number of 5 cards combinations out of a deck of 52 cards if there is exactly one ace in each combination is

(a) 768320 (b) 777320
(c) 778230 (d) 778320

8. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(a) $3a^2 - 26a + 55 = 0$ (b) $3a^2 - 32a + 84 = 0$
(c) $3a^2 - 34a + 91 = 0$ (d) $3a^2 - 23a + 44 = 0$

9. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A}\log_e\left(\frac{f(x)}{g(x)}\right) + c$, then

(a) $A = \sin(a - b)$ (b) $f(x) = \sec(x - b)$
(c) $g(x) = \sec(x - a)$ (d) $A = \sin(x - a)$

10. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are

(a) 1 (b) 2 (c) -1 (d) -2

11. If $f(x) = x^3 - x^2 + 100x + 2002$, then

(a) $f(1000) > f(1001)$

(b) $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

(c) $f(x - 1) > f(x - 2)$

(d) none of these

12. Consider the points $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$, where $abc \neq 0$; then

(a) the equation of plane ABC is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(b) the area of ΔABC is $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$

(c) the equation of the plane ABC is $ax + by + cz = 1$

(d) none of these

13. The solution of $\frac{xdx+ydy}{xdy-ydx}=\sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$, is equal to

(a) $\sqrt{x^2+y^2}=a\left(\sin\left(\tan^{-1}\frac{y}{x}\right)+c\right)$

(b) $\sqrt{x^2+y^2}=a\left(\cos\left(\tan^{-1}\frac{y}{x}\right)+c\right)$

(c) $\sqrt{x^2+y^2}=a\left(\tan\left(\sin^{-1}\frac{y}{x}\right)+c\right)$

(d) $y=x\tan\left(\sin^{-1}\left(\frac{1}{a}\sqrt{x^2+y^2}\right)-c\right)$

14. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=$

(a) $2(\vec{a}\times\vec{b})$ (b) $6(\vec{b}\times\vec{c})$

(c) $3(\vec{c}\times\vec{a})$ (d) $\vec{0}$

15. If the equation $ax^2 + bx + c = 0$ $(a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(a) $b^2 - 4ac > 0$ (b) $c < 0$

(c) $a + |b| + c < 0$ (d) $4a + 2|b| + c < 0$

16. $7^9 + 9^7$ is divisible by

(a) 128 (b) 24 (c) 64 (d) 72

17. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9, is

(a) 0 (b) 2 (c) 7 (d) 8

18. If in ΔABC, $\begin{vmatrix}\sin A & \sin B & \sin C\\ \sin B & \sin C & \sin A\\ \sin C & \sin A & \sin B\end{vmatrix}=0$, then

(a) ΔABC is equilateral

(b) $a^3 + b^3 + c^3 = 3abc$

(c) $\Sigma\sin A = 0$

(d) $\Sigma\sin^3 A - 3\,\Pi\sin A = 0$

19. If $\alpha \leq \sin^{-1}x + \cos^{-1}x + \tan^{-1}x < \beta$, then

(a) $\alpha = 0$ (b) $\beta=\frac{\pi}{2}$ (c) $\alpha=\frac{\pi}{4}$ (d) $\beta=\frac{3\pi}{4}$

20. If (α, α^2) lies inside the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$, $5x - 6y - 1 = 0$, then

(a) $2\alpha + 3\alpha^2 - 1 > 0$ (b) $\alpha + 2\alpha^2 - 3 > 0$

(c) $\alpha + 2\alpha^2 - 3 < 0$ (d) $6\alpha^2 - 5\alpha + 1 > 0$

Comprehension Type

Paragraph for Q. No. 21 to 23

Consider the quadratic polynomial
$f(x) = x^2 - 4ax + 5a^2 - 6a$, $(a \in R)$.

21. The value of a for which roots of $f(x) = 0$ are equal in magnitude and opposite in sign, is

(a) 0 (b) 1

(c) 2 (d) none of these

22. Number of values of a for which the equation $f(x) = 0$ has one root equal to zero, is

(a) 0 (b) 1 (c) 2 (d) 3

23. The largest integral value of 'a' for which range of $f(x)$ is $[-5, \infty)$ for every real x, is

(a) 1 (b) 5 (c) 7 (d) 10

Paragraph for Q. No. 24 to 26

Consider the lines $L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$,
$L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

24. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

25. The shortest distance between L_1 and L_2 is

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

26. The distance of the point (1, 1, 1) from the plane passing through the point $(-1, -2, -1)$ and whose normal is perpendicular to both L_1 and L_2 is

(a) $\frac{2}{5\sqrt{3}}$ (b) $\frac{7}{5\sqrt{3}}$ (c) $\frac{13}{5\sqrt{3}}$ (d) $\frac{23}{5\sqrt{3}}$

Paragraph for Q. No. 27 and 28

Let $\frac{b\cos x}{2\cos 2x-1}=\frac{b+\sin x}{(\cos^2 x-3\sin^2 x)\tan x}$, $b\in R$

27. The equation has solutions if

(a) $b\in\left(-\infty,\frac{1}{2}\right)-\left\{-1,0,\frac{1}{3}\right\}$

(b) $b\in(-\infty,1)-\left\{-1,0,\frac{1}{3}\right\}$

(c) $b\in R-\left\{-1,0,\frac{1}{3}\right\}$ (d) none of these

28. For any value of b for which the equation has a solution, the number of solutions when $x \in (0, 2\pi)$ are always

(a) infinite

(b) depends upon the value of b

(c) two

(d) none of these

Matrix Match Type

29. Match the following.

	Column-I		Column-II
(A)	If sum of the coefficient of the first, second and third terms in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is 46, then coefficient of the term that does not contain x is	(P)	3
(B)	For a positive integer n, if the expansion of $\left(\frac{5}{x^2}+x^4\right)^n$ has a term independent of x, then minimum value of n can be	(Q)	15
(C)	If in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$, sum of the coefficients of x^5 and x^{10} is zero, then the value of n is	(R)	19
(D)	The coefficient of x^4 in the expansion of $(1 + x + x^2)^4$ is	(S)	84

30. n whole numbers are randomly chosen and multiplied. Then match the following columns :

	Column-I		Column-II
(A)	The probability that the last digit is 1, 3, 7 or 9 is	(P)	$\frac{8^n-4^n}{10^n}$
(B)	The probability that the last digit is 2, 4, 6, 8 is	(Q)	$\frac{5^n-4^n}{10^n}$
(C)	The probability that the last digit is 5 is	(R)	$\frac{4^n}{10^n}$
(D)	The probability that the last digit is zero is	(S)	$\frac{10^n-8^n-5^n+4^n}{10^n}$

31. Match the greatest and least value of functions in column-I with column-II.

	Column-I		Column-II
(A)	$x + 2\sqrt{x}$ on [0, 4]	(P)	13 and 4
(B)	$\frac{x-1}{x+1}$ on [0, 4]	(Q)	8 and 0
(C)	$x^4 - 2x^2 + 5$ on [–2, 2]	(R)	3/5 and –1

Numerical Answer Type

32. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ________.

33. $f(x) = 1 - x^3 - x^4 - 2x^5 = g(x) + h(x)$, where g is an even function and h is an odd function. If $h(-5) = 125\,k$, then k equals ________.

34. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is ________.

35. The value of $\lim\limits_{x\to\pi/4} \frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$ is ______.

36. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$ is $\frac{1}{x^5y^5}=\frac{p}{qx^2}+c$, then the value of $p + q$ is ________.

37. Let $F(x) = \int_x^{x+3} 2t(5-t)dt$. If M is the maximum value of $F(x)$, then the sum of the digits of M is ________.

38. If A is the area bounded by $y = x^{2/3} - x^{4/3}$ and x-axis, then the value of $35A/4$ is ________.

39. The greatest possible number of points of intersection of 8 straight lines and 4 circles is ________.

40. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then $16\,P(X \geq 1) =$ ________.

SOLUTIONS

1. (c) : Standard equation of hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Also, $e^2=\left(1+\frac{b^2}{a^2}\right) \Rightarrow 2^2-1=\frac{b^2}{a^2}$

[Given, eccentricity = 2]

$\Rightarrow \quad b^2 = 3a^2$

Since, hyperbola passes through the point (4, 6).

$\therefore \quad \frac{16}{a^2}-\frac{36}{b^2}=1 \Rightarrow \frac{16\times 3-36}{3a^2}=1 \quad [\because b^2=3a^2]$

$\Rightarrow \quad 3a^2 = 12 \quad \therefore \ a^2 = 4 \ \Rightarrow \ b^2 = 12$

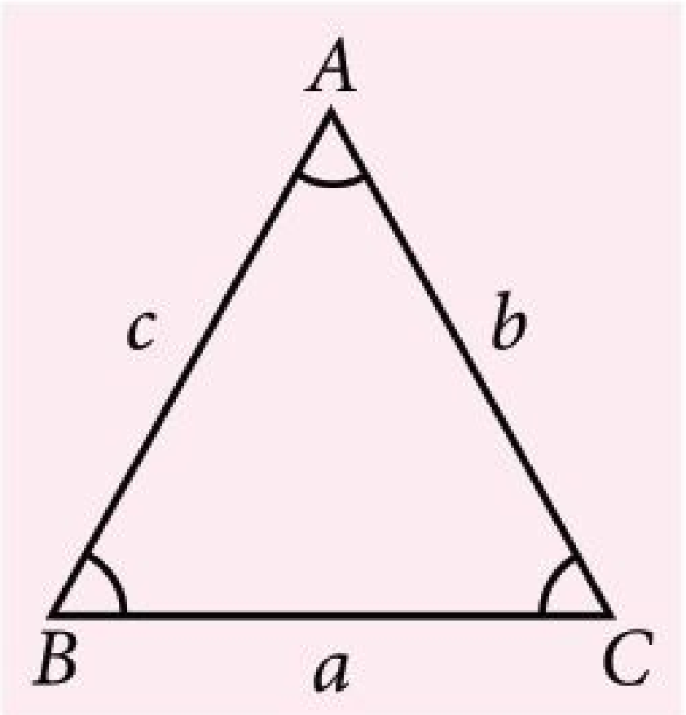
Now, equation of the tangent to the hyperbola at(4, 6) is

$$\frac{x\cdot 4}{4}-\frac{y\cdot 6}{12}=1$$

$\Rightarrow \quad x-\frac{y}{2}=1 \ \Rightarrow \ 2x-y-2=0$

2. (a) : Let a, b, c be the lengths of the sides of the triangle ABC.

Let $a < b < c \Rightarrow \angle C = 2\angle A$ (Given)



$\Rightarrow \quad \sin C = \sin 2A$

$\Rightarrow \quad \sin C = 2 \sin A \cos A$

$\Rightarrow \quad \frac{\sin C}{\sin A}=2\cos A$

$\Rightarrow \quad \frac{c}{a}=2\left(\frac{b^2+c^2-a^2}{2bc}\right)$

[Using sine and cosine formula]

$\Rightarrow \quad bc^2 = a(b^2 + c^2 - a^2)$ ...(i)

Putting $a = b - \lambda$ and $c = b + \lambda$, $\lambda > 0$ in (i), we get

$b(b + \lambda)^2 = (b - \lambda)\,[b^2 + (b + \lambda)^2 - (b - \lambda)^2]$

[$\because$ a, b, c are in A.P.]

$\Rightarrow \quad b(b^2 + \lambda^2 + 2b\lambda) = (b - \lambda)(b^2 + 4b\lambda)$

$\Rightarrow \quad b^2 + \lambda^2 + 2b\lambda = b^2 + 3b\lambda - 4\lambda^2 \ \Rightarrow \ 5\lambda^2 = b\lambda$

$\Rightarrow \quad \lambda=\frac{b}{5}$

$\therefore \quad a=b-\frac{b}{5}=\frac{4b}{5}$ and $c=b+\frac{b}{5}=\frac{6b}{5}$

Hence, the required ratio is 4 : 5 : 6.

3. (c) : The given equation can be written as

$(z^5 - 32)(z^5 + 31) = 0$

$\Rightarrow \quad z = 2(1)^{1/5}$ or $z=\sqrt[5]{31}(-1)^{1/5}$

4. (a) : If z lies on the line, then

$$\arg\left(\frac{z-ib}{z-a}\right)=0 \text{ or } \pi \quad \Rightarrow \quad \frac{z-ib}{z-a}=\frac{\bar{z}+ib}{\bar{z}-a}$$

$\Rightarrow \quad z(-a-ib)+\bar{z}(-ib+a)+2iab=0$

$\Rightarrow \quad z\left(-\frac{1}{2ib}-\frac{1}{2a}\right)+\bar{z}\left(-\frac{1}{2a}+\frac{1}{2ib}\right)+1=0$

$\Rightarrow \quad z\left(\frac{1}{2a}+\frac{1}{2ib}\right)+\bar{z}\left(\frac{1}{2a}-\frac{1}{2ib}\right)=1$

$\Rightarrow \quad z\left(\frac{1}{2a}-\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}+\frac{i}{2b}\right)=1$

5. (a, b, c) : We have, $\alpha=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}$

and $1+\alpha+\alpha^2+\alpha^3+\alpha^4=0$

$\therefore \quad |1+\alpha+\alpha^2+\alpha^3| = |-\alpha^4| = |\alpha|^4=1$

Also, $|1+\alpha+\alpha^2| = |-\alpha^3(1+\alpha)| = |1+\alpha|$ ... (i)

Now, $|1 + \alpha|$

$=\left|1+\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right|=\left|2\cos\frac{\pi}{5}\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\right|$

$=2\cos\frac{\pi}{5}$

Again from (i), $|1+\alpha| = |1+\alpha+\alpha^2| = 2\cos\frac{\pi}{5}$.

6. (c) : Let $I=\int_0^1\frac{8\ln(1+x)}{1+x^2}dx$

Let $J=\int_0^1\frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta$

$\therefore \quad J=\int_0^{\pi/4}\ln(1+\tan\theta)d\theta$ ...(i)

Also, $J=\int_0^{\pi/4}\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$ ...(ii)

Adding (i) and (ii), we get

$$2J=\int_0^{\pi/4}\left[\ln(1+\tan\theta)+\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right]d\theta$$

$\Rightarrow \quad 2J=\int_0^{\pi/4}\ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$\Rightarrow \quad 2J=\int_0^{\pi/4}(\ln 2)d\theta=\frac{\pi}{4}\ln 2 \ \Rightarrow \ 8J=\pi\ln 2$

$\Rightarrow \quad I = 8J = \pi\ln 2$

7. (d): Total number of cards = 52

Number of ace cards = 4

Number of non-ace cards = 48

$\therefore$ One ace card out of 4 can be selected in ${}^4C_1$ ways.

Remaining 4 cards out of 48 cards can be selected in ${}^{48}C_4$ ways.

$\therefore$ Required number of ways = ${}^4C_1 \times {}^{48}C_4$

$$=\frac{4!}{3!\times 1!}\times\frac{48!}{4!\times 44!}=\frac{48\times 47\times 46\times 45}{6}=778320$$

8. (b): Since, $\text{S.D.}=\sqrt{\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2}$

Now, $\frac{49}{4}=\frac{4+9+a^2+121}{4}-\left(\frac{16+a}{4}\right)^2$

$\Rightarrow\quad 49\times 4=4(134+a^2)-(256+32a+a^2)$

$\Rightarrow\quad 3a^2-32a+84=0$

9. (a, b, c) : $I=\frac{1}{\sin(a-b)}\int\frac{\sin(x-b-(x-a))dx}{\cos(x-a)\cos(x-b)}$

$$=\frac{1}{\sin(a-b)}\int(\tan(x-b)-\tan(x-a))dx$$

$$=\frac{1}{\sin(a-b)}\left[\log_e(\sec(x-b))-\log_e(\sec(x-a))\right]+c$$

$$=\frac{1}{\sin(a-b)}\left[\log_e\left(\frac{\sec(x-b)}{\sec(x-a)}\right)\right]+c$$

10. (a, b) : $\overrightarrow{BC}=(a\hat{i}-3\hat{j}+\hat{k})-(\hat{i}-3\hat{j}-5\hat{k})$

$=(a-1)\hat{i}+6\hat{k}$

$\overrightarrow{CA}=(2\hat{i}-\hat{j}+\hat{k})-(a\hat{i}-3\hat{j}+\hat{k})=(2-a)\hat{i}+2\hat{j}$

Now, $\overrightarrow{BC}\cdot\overrightarrow{CA}=0\Rightarrow(a-1)(2-a)=0$

$\therefore\quad a=1,2$

11. (b, c) : Here, $f(x)=x^3-x^2+100x+2002$

$\Rightarrow\quad f'(x)=3x^2-2x+100>0\ \forall\ x\in R$

$\therefore\quad f(x)$ is strictly increasing.

Therefore, $f\left(\frac{1}{2000}\right)>f\left(\frac{1}{2001}\right)$

Also, $f(x-1)>f(x-2)$ as $x-1>x-2\ \forall\ x\in R$

12. (a, b) : The three points lie in the plane

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(intercept form) and area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$=\frac{1}{2}|(-a\hat{i}+b\hat{j})\times(-a\hat{i}+c\hat{k})|$$

$$=\frac{1}{2}|\hat{i}(bc)-\hat{j}(-ac)+\hat{k}(ab)|$$

$$=\frac{1}{2}\sqrt{b^2c^2+a^2c^2+a^2b^2}$$

13. (a,d) : Taking $x=r\cos\theta$ and $y=r\sin\theta$, so that $x^2+y^2=r^2$ and $y/x=\tan\theta$, we have $xdx+ydy=rdr$ and $xdy-ydx=x^2\sec^2\theta d\theta=r^2d\theta$.

The given equation can be transformed into

$$\frac{rdr}{r^2d\theta}=\sqrt{\frac{a^2-r^2}{r^2}}\Rightarrow\frac{dr}{d\theta}=\sqrt{a^2-r^2}$$

$$\Rightarrow\quad\frac{dr}{\sqrt{a^2-r^2}}=d\theta$$

Integrating both sides, we get $\sin^{-1}\left(\frac{r}{a}\right)=\theta+c$

$$\Rightarrow\quad\sin^{-1}\left(\frac{\sqrt{(x^2+y^2)}}{a}\right)=\tan^{-1}\left(\frac{y}{x}\right)+c\qquad\text{...(i)}$$

$$\Rightarrow\quad\sqrt{x^2+y^2}=a\sin\{\tan^{-1}(y/x)+c\}$$

From (i), $\tan^{-1}\left(\frac{y}{x}\right)=\sin^{-1}\left(\frac{\sqrt{(x^2+y^2)}}{a}\right)-c$

$$\Rightarrow\quad y=x\tan\left\{\sin^{-1}\left(\frac{\sqrt{(x^2+y^2)}}{a}\right)-c\right\}$$

14. (a,b,c) : We know that

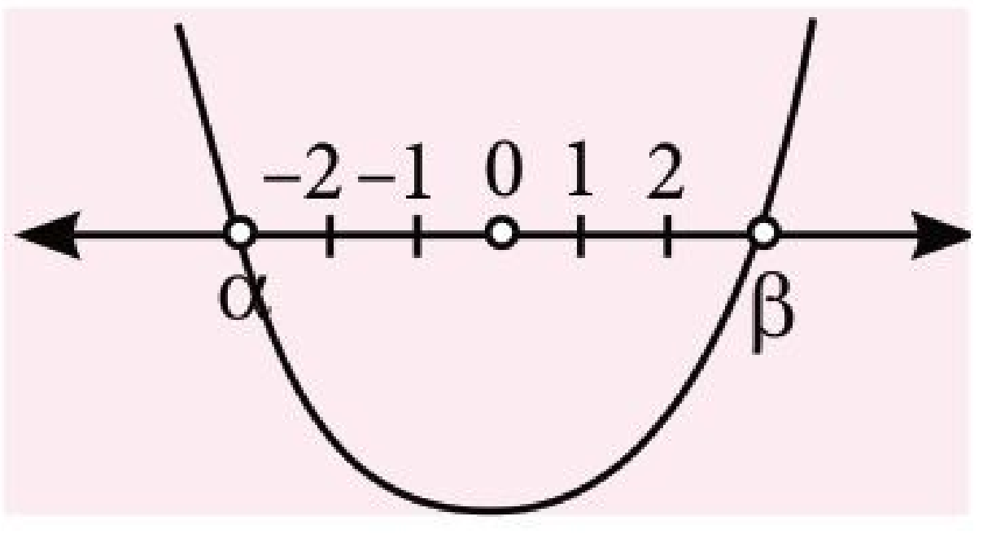
$\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Given, $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$,

$\Rightarrow\quad 2(\vec{a}\times\vec{b})=6(\vec{b}\times\vec{c})=3(\vec{c}\times\vec{a})$

Hence, $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=2(\vec{a}\times\vec{b})$

or $\quad 6(\vec{b}\times\vec{c})$ or $3(\vec{c}\times\vec{a})$.



15. (a,b,c,d) : Since, the equation has two distinct roots α and β, the discriminant $b^2-4ac>0$, we must have $f(x)=ax^2+bx+c<0$ for $\alpha<x<\beta$

Since, $\alpha<0<\beta$ we must have $f(0)=c<0$

Also, as $\alpha<-1$, $1<\beta$,

we get $f(-1)=a-b+c<0$ and $f(1)=a+b+c<0$,

$\Rightarrow\quad a+|b|+c<0$

Since, $\alpha<-2$, $2<\beta$, we must have

$f(-2)=4a-2b+c<0$ and $f(2)=4a+2b+c<0$

$\Rightarrow\quad 4a+2|b|+c<0$

16. (c) : We have, $7^9+9^7=7^2(8-1)^7+(8+1)^7$

$$=7^2\sum_{r=0}^{7}{}^7C_r8^{7-r}(-1)^r+\sum_{r=0}^{7}{}^7C_r8^{7-r}$$

$$=\sum_{r=0}^{7}{}^7C_r8^{7-r}\left\{49(-1)^r+1\right\}$$

$$=\sum_{r=0}^{3}\left\{{}^7C_{2r}\times 8^{7-2r}\times 50-{}^7C_{2r+1}\times 8^{7-2r-1}\times 48\right\}$$

$$=\sum_{r=0}^{3}({}^7C_{2r}\times 50-{}^7C_{2r+1}\times 6)8^{7-2r}$$

$= (50 - 42) \times 8^7 + (21 \times 50 - 35 \times 6) \times 8^5$
$\quad + (35 \times 50 - 21 \times 6) \times 8^3 + (7 \times 50 - 6) \times 8$
$= 8^8 + 105 \times 8^6 + 203 \times 8^4 + 43 \times 8^2$
$= 8^2 (8^6 + 105 \times 8^4 + 203 \times 8^2 + 43)$

17. (b): We have, $8^{2n} - (62)^{2n+1}$
$= (63 + 1)^n - (63 - 1)^{2n+1} = (1 + 63)^n + (1 - 63)^{2n+1}$
$= \left\{1 + {}^nC_1 \times 63 + {}^nC_2 \times (63)^2 + \ldots + {}^nC_n \times (63)^n\right\}$
$+ \{1 - {}^{2n+1}C_1 \times 63 + {}^{2n+1}C_2 \times (63)^2 + \ldots + (-1)^{2n+1}(63)^{2n+1}\}$
$= 2 + 63\,\{{}^nC_1 + {}^nC_2 \times 63 + \ldots + {}^nC_n \times (63)^{n-1}\}$
$+ \{-{}^{2n+1}C_1 + {}^{2n+1}C_2 \times 63 + \ldots + (-1)^{2n+1}(63)^{2n}\} \times 63$
$= 2 +$ a multiple of 63
$\therefore$ Required remainder = 2.

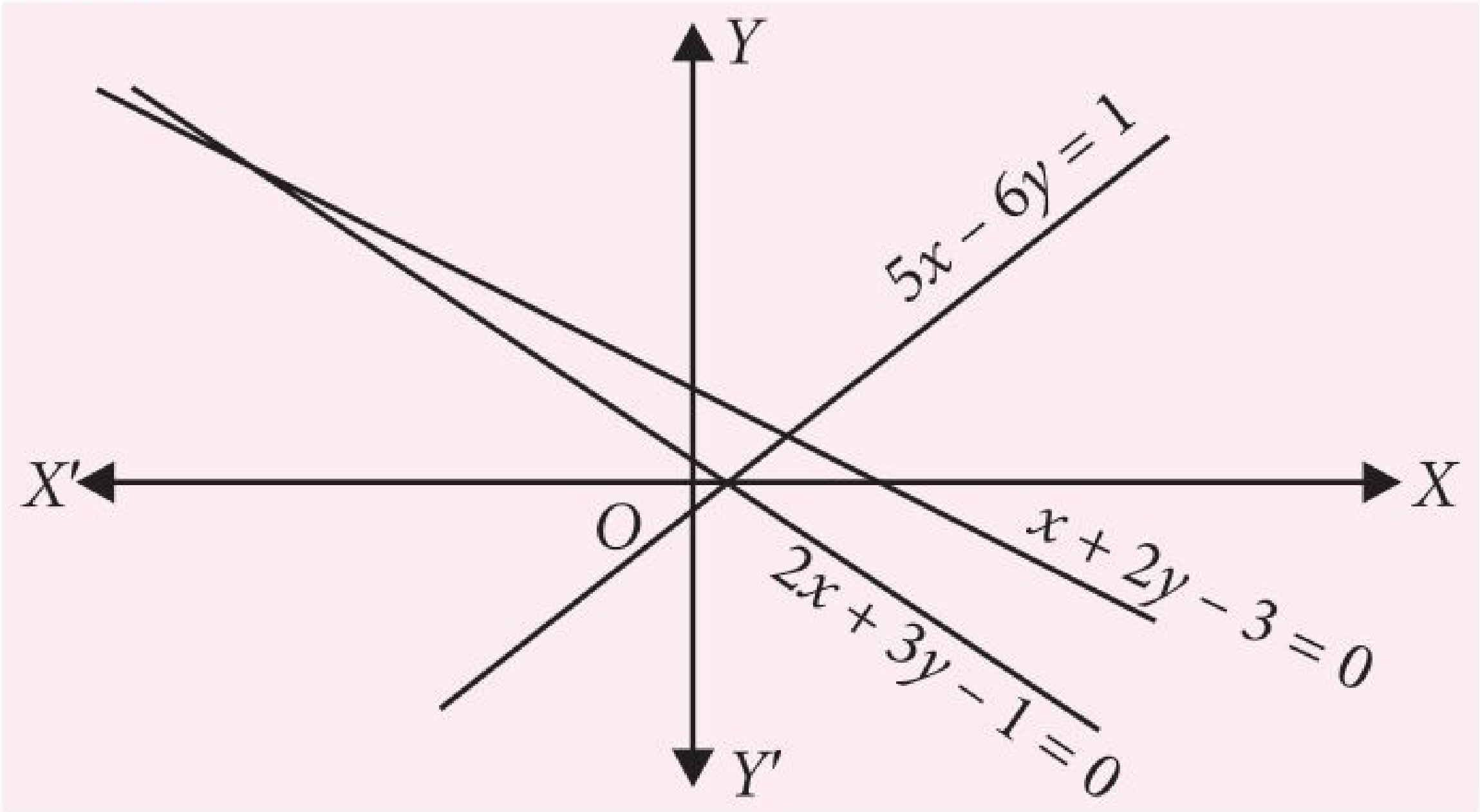
18. (a, b,d)

19. (c,d) : $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = \dfrac{\pi}{2} + \tan^{-1} x$

Since $x \in [-1, 1] \ \therefore\ -\dfrac{\pi}{4} \le \tan^{-1} x \le \dfrac{\pi}{4}$

$\Rightarrow \dfrac{\pi}{4} \le \dfrac{\pi}{2} + \tan^{-1} x \le \dfrac{3\pi}{4} \ \Rightarrow\ \alpha = \dfrac{\pi}{4}, \beta = \dfrac{3\pi}{4}$

20. (a,c,d):



The point (α, α^2) lies above the line $2x + 3y - 1 = 0$
Hence, $2\alpha + 3\alpha^2 - 1 > 0$
The point (α, α^2) lies below the line $x + 2y - 3 = 0$.
Hence, $\alpha + 2\alpha^2 - 3 < 0$
The point (α, α^2) lies above the line $5x - 6y - 1 = 0$.
Hence, $5\alpha - 6\alpha^2 - 1 < 0$ or $6\alpha^2 - 5\alpha + 1 > 0$

21. (d) : Since, roots of $f(x) = 0$ are equal in magnitude and opposite in sign.
$\therefore$ Sum of the roots = 0 and product of the roots < 0
Sum of the roots = 0 $\Rightarrow a = 0$ ...(i)
Product of the roots < 0 $\Rightarrow 5a^2 - 6a < 0$

$\Rightarrow \quad a \in \left(0, \dfrac{6}{5}\right)$ ...(ii)

From (i) and (ii), $a = \phi$

22. (b): If one root of $f(x) = 0$ is zero, then product of the roots is 0.

$\Rightarrow \quad 5a^2 - 6a = 0 \ \Rightarrow\ a = 0, \dfrac{6}{5}$

Hence, $a = \dfrac{6}{5}$ ($\because$ $a \neq 0$ as sum of the roots is non zero.)

23. (b): Range of $f(x)$ is $[-5, \infty)$ for every real x or range of $f(x) + 5$ is $[0, \infty)$ for every real x.
Now $f(x) + 5 = x^2 - 4ax + 5a^2 - 6a + 5$
For range to be $[0, \infty)$, $D = 0$
$\Rightarrow \quad 16a^2 - 4(5a^2 - 6a + 5) = 0$
$\Rightarrow \quad a^2 - 6a + 5 = 0 \Rightarrow a = 5$ or 1

24. (b): The vector perpendicular to L_1 and L_2 is given by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$\therefore$ Required unit vector $(\hat{n})$ is $\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

25. (d): The points on L_1 and L_2 are $A(-1, -2, -1)$ and $C(2, -2, 3)$
The shortest distance between L_1 and L_2 is

$$\overrightarrow{AC} \cdot \hat{n} = (3\hat{i} + 4\hat{k}) \cdot \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

26. (c) : The d.r's of the normal is <–1, –7, 5 >
$\therefore$ The plane is $-(x + 1) - 7(y + 2) + 5(z + 1) = 0$
$\Rightarrow \quad x + 7y - 5z + 10 = 0$
The distance of (1, 1, 1) is $13/\sqrt{75} = 13/5\sqrt{3}$

(27-28): We have, $\dfrac{b\cos x}{2\cos 2x - 1}$

$$= \frac{b + \sin x}{(\cos^2 x - 3\sin^2 x)\tan x} \quad \text{...(i)}$$

(i) $2\cos 2x - 1 \neq 0 \Rightarrow x \neq n\pi \pm \dfrac{\pi}{6}$

(ii) $\tan x \neq 0 \Rightarrow x \neq n\pi$

(ii) $\cos^2 x - 3\sin^2 x \neq 0 \Rightarrow x \neq n\pi \pm \dfrac{\pi}{6}$

Also, $2\cos 2x - 1 = 2(\cos^2 x - \sin^2 x) - (\cos^2 x + \sin^2 x)$
$= \cos^2 x - 3\sin^2 x$

$\therefore$ (i) reduces to $b \sin x = b + \sin x \Rightarrow \sin x = \dfrac{b}{b-1}$
Now, $-1 \le \sin x \le 1$

$\Rightarrow \quad -1 \le \dfrac{b}{b-1} \le 1 \Rightarrow \dfrac{b}{b-1} + 1 \ge 0$ and $\dfrac{b}{b-1} - 1 \le 0$

$\Rightarrow \quad \dfrac{2b-1}{b-1} \ge 0$ and $\dfrac{1}{b-1} \le 0$

$\Rightarrow \quad b \in \left(-\infty, \dfrac{1}{2}\right] \cup (1, \infty)$ and $b \in (-\infty, 1)$

Now, $\sin x \neq 0, \pm\dfrac{1}{2}$ or $b \neq 0, -1, \dfrac{1}{3}$

When $b=\frac{1}{2}$, then $\sin x = -1$, which is not possible, as $\tan x$ is not defined for this value of $\sin x$.

So, $b\in\left(-\infty,\frac{1}{2}\right)-\left\{-1,0,\frac{1}{3}\right\}$

For any other value of b, $\sin x$ takes two values for $x\in(0, 2\pi)$.

27. (a) **28. (c)**

29. (A) → (S) ; (B) → (P) ; (C) → (Q) ; (D) → (R)

(A) Given, ${}^nC_0 + {}^nC_1 + {}^nC_2 = 46$

$\Rightarrow \quad 1+n+\frac{n(n-1)}{2}=46$

$\Rightarrow \quad 2 + 2n + n^2 - n = 92 \Rightarrow n^2 + n - 90 = 0$

$\Rightarrow \quad (n - 9)(n + 10) = 0 \Rightarrow n = 9, n \neq -10$

$\therefore \quad \left(x^2+\frac{1}{x}\right)^n=\left(x^2+\frac{1}{x}\right)^9$

$\therefore \quad T_{r+1}={}^9C_r(x^2)^{9-r}.\left(\frac{1}{x}\right)^r={}^9C_r\cdot x^{18-3r}$

Now, $18 - 3r = 0 \Rightarrow r = 6$

$\Rightarrow \quad T_{6+1}={}^9C_6={}^9C_3=\frac{9\cdot8\cdot7}{1\cdot2\cdot3}=84$

(B) $T_{r+1}={}^nC_r\left(\frac{5}{x^2}\right)^{n-r}.(x^4)^r$

$= {}^nC_r \cdot (5)^{n-r} \cdot x^{-2n+2r+4r}$

$= {}^nC_r \cdot (5)^{n-r} \cdot x^{6r-2n}$

$\because \quad 6r - 2n = 0 \quad \Rightarrow r = n/3 \quad \therefore \quad n = 3, 6, 9, ...$

(C) $\because \; T_{r+1}={}^nC_r(x^3)^{n-r}\cdot\left(-\frac{1}{x^2}\right)^r={}^nC_r\cdot(-1)^r\cdot x^{3n-5r}$

For coefficient of x^5, we get $3n - 5r = 5$

$\Rightarrow \quad r=\frac{3n-5}{5}=p(\text{say})$

For coefficient of x^{10}, we get $3n - 5r = 10$

$\Rightarrow \quad r=\frac{3n-10}{5}=q(\text{say})$

$\because \quad p - q = 1$ (using values of r) ...(i)

Also, ${}^nC_p(-1)^p + {}^nC_q(-1)^q = 0$

$\Rightarrow \quad {}^nC_p(-1)^p + {}^nC_{p-1}(-1)^{p-1} = 0$ (From (i))

$\Rightarrow \quad (-1)^p\{{}^nC_p - {}^nC_{p-1}\} = 0$

$\Rightarrow {}^nC_p - {}^nC_{p-1} = 0$

$\Rightarrow \quad {}^nC_p = {}^nC_{p-1} \quad \Rightarrow n = p + p - 1 \Rightarrow p=\frac{n+1}{2}$

$\therefore \quad r = p \quad \Rightarrow \frac{3n-5}{5}=\frac{n+1}{2}$

$\Rightarrow \quad 6n - 10 = 5n + 5 \quad \Rightarrow n = 15$

(D) We have, $(1 + x + x^2)^4$

$= (1 + x(1 + x))^4 = 1 + {}^4C_1\, x(1 + x) + {}^4C_2\, x^2(1 + x)^2 + {}^4C_3\, x^3(1 + x)^3 + {}^4C_4\, x^4(1 + x)^4$

$= 1 + 4x(1 + x) + 6x^2(1 + x^2 + 2x) + 4x^3 (1 + x^3 + 3x + 3x^2) + x^4(x^4 + 4x^3 + 6x^2 + 4x + 1)$

$\therefore$ Coefficient of $x^4 = 6 + 12 + 1 = 19$

30. (A) → (R) ; (B) → (P) ; (C) → (Q); (D) → (S)

(A) The required event will occur if last digit in all the chosen numbers is 1, 3, 7 or 9. Therefore, the required probability is $(4/10)^n$.

(B) The required probability (that the last digit is 2, 4, 6, 8) = P (last digit is 1, 2, 3, 4, 6, 7, 8, 9) – P (last digit is 1, 3, 7, 9)

$=\frac{8^n-4^n}{10^n}$

(C) P(Last digit is 1, 3, 5, 7, 9) – P(Last digit is 1, 3, 7, 9)

$=\frac{5^n-4^n}{10^n}$

(D) The required probability is

P(Last digit is 0, 5) – P(Last digit is 5)

$=\frac{(10^n-8^n)-(5^n-4^n)}{10^n}=\frac{10^n-8^n-5^n+4^n}{10^n}$

31. (A) → (Q) ; (B) → (R) ; (C) → (P)

(A) Let $f(x) = x + 2\sqrt{x}$, then

$f'(x) = 1 + x^{-1/2} > 0 \; \forall \; x \in [0, 4]$.

Thus, the least value of $f(x)$ is $f(0) = 0$ and the greatest value is $f(4) = 4 + 2\sqrt{4} = 8$.

(B) Let $f(x) = \frac{x-1}{x+1}$, then $f'(x) = \frac{2}{(x+1)^2} > 0$ for any $x \neq -1$. Thus, the least value of $f(x) = f(0) = -1$ and the greatest value of $f(x) = f(4) = 3/5$.

(C) Let $f(x) = x^4 - 2x^2 + 5$

Then, $f'(x) = 4x^3 - 4x$

Now, $f'(x) = 0 \Rightarrow 4x^3 - 4x = 0$

$\Rightarrow \quad 4x(x^2 - 1) = 0 \Rightarrow x = 0, 1, -1$

Now, $f(0) = 5, f(1) = 4 = f(-1), f(-2) = f(2) = 13$.

Thus, greatest value of $f(x)$ is 13 and least value of $f(x)$ is 4.

32. (5) : The pair of lines is

$3x^2 - xy + 3y^2 + (2x - 3y)\frac{(2x+3y)}{k}-4\frac{(2x+3y)^2}{k^2}=0$

Now, coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow\ 3+3+\frac{4-9}{k}-\frac{4(4+9)}{k^2}=0$$

$$\Rightarrow 6k^2-5k-52=0 \quad \Rightarrow \quad 6k-\frac{52}{k}=5.$$

33. (51) : $f(x) = 1 - x^3 - x^4 - 2x^5 = (1 - x^4) - x^3 - 2x^5$
$= g(x) + h(x)$

$\therefore \quad h(x) = -x^3 - 2x^5$

$\therefore \quad h(-5) = 6375 = 125\,k$

$\therefore \quad k = 51$

34. (2250) : Section A (5) Section B (5) Section C (5)

$$\left.\begin{matrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{matrix}\right\} \text{Selection of questions}$$

$\therefore$ Total number of ways

$= 3({}^5C_1\,{}^5C_2\,{}^5C_2) + 3({}^5C_1\,{}^5C_1\,{}^5C_3)$

$= 3(10 \times 10 \times 5) + 3\,(5 \times 5 \times 10)$

$= 1500 + 750 = 2250$

35. (10) : $\displaystyle\lim_{x\to\pi/4}\frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$

$$=\sqrt{2}\lim_{x\to\pi/4}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$$

$$=\sqrt{2}\lim_{x\to\pi/4}\frac{2^{5/2}-((\sin x+\cos x)^2)^{5/2}}{1-\sin 2x}$$

$$=\sqrt{2}\lim_{x\to\pi/4}\frac{2^{5/2}-(1+\sin 2x)^{5/2}}{2-(1+\sin 2x)}$$

$$=\sqrt{2}\lim_{u\to 2}\frac{2^{5/2}-u^{5/2}}{2-u}=\sqrt{2}\,\frac{5}{2}.2^{3/2} \qquad (\because\ u = 1 + \sin 2x)$$

$= 10$

36. (7) : We have, $\dfrac{dy}{dx}+\dfrac{y}{x}=x^2y^6 \Rightarrow \dfrac{1}{y^6}\dfrac{dy}{dx}+\dfrac{1}{xy^5}=x^2$

Put $\dfrac{1}{y^5}=z \Rightarrow \dfrac{-5}{y^6}\dfrac{dy}{dx}=\dfrac{dz}{dx}$

$\therefore \quad -\dfrac{1}{5}\left(\dfrac{dz}{dx}\right)+\dfrac{z}{x}=x^2 \Rightarrow \text{I.F.}=\dfrac{1}{x^5}$

Hence solution is $\dfrac{1}{x^5y^5}=\dfrac{5}{2x^2}+c$

$\therefore \quad p + q = 5 + 2 = 7$

37. (6) : $F'(x) = 2(x + 3)(5 - x - 3) - 2x\,(5 - x)$
$= 12(1 - x)$

$F'(x) = 0 \quad \Rightarrow \quad x = 1$

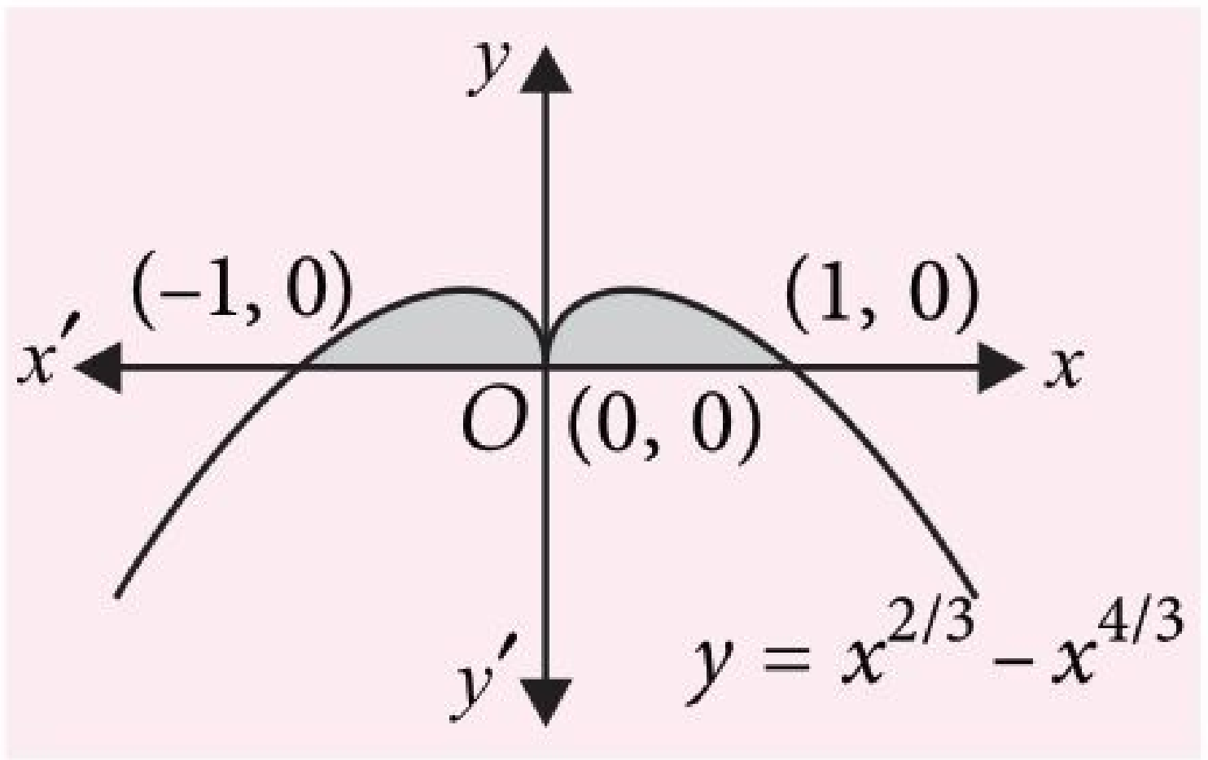
So, $\displaystyle M=\int_1^4 2t(5-t)\,dt=33$

$\therefore$ Required sum = 6

38. (3) : We have $y = f(x) = x^{2/3} - x^{4/3}$

It meets the x-axis at $x = 0, \pm 1$.

Also, function is even.



$\therefore$ Required area,

$$A=2\left[\int_0^1 (x^{2/3}-x^{4/3})\,dx\right]$$

$$=2\left[\frac{3x^{5/3}}{5}-\frac{3x^{7/3}}{7}\right]_0^1=\frac{12}{35}$$

$\therefore \quad \dfrac{35A}{4}=3$

39. (104) : The required number of points

$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + ({}^8C_1 \times {}^4C_1) \times 2$

$= 28 + 12 + 32 \times 2 = 104$

40. (15) : $np = 2$ ...(i), $npq = 1$ ...(ii)

$\Rightarrow \quad q=\dfrac{1}{2},\ p=\dfrac{1}{2},\ n=4$ (from (i) and (ii))

$\therefore \quad P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$=1-{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4=1-\frac{1}{16}=\frac{15}{16}$$

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beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. If $S_{(P)} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{P} \, (P \in N)$, then $S_{(1)} + S_{(2)} + S_{(3)} + \ldots + S_{(P-1)}$ is

(a) $(P - 1)\, S_{(P)} + P$ (b) $P\, S_{(P-1)} + 1 - P$

(c) $(P - 1)\, S_{(P-1)} + 1$ (d) $P\, S_{(P)} - 1$

2. The total number of permutations of n different things taken not more than r at a time, when each things may be repeated any number of times, is

(a) $n^r - 1$ (b) $\frac{n(n^r - 1)}{n - 1}$

(c) $\frac{n^r - 1}{n - 1}$ (d) $\frac{n(n^r + 1)}{n + 1}$

3. Let $f(n)$ denote the square of the sum of the digits of natural number n & $f^2(n)$ denotes $f(f(n))$, $f^3(n)$ denotes $f(f(f(n)))$ and so on, then the value of $\frac{f^{2021}(2021) - f^{2020}(2021)}{f^{2023}(2021) - f^{2022}(2021)}$ equals

(a) 3 (b) 2 (c) 1 (d) 0

4. If $\left(m_i, \frac{1}{m_i}\right)$, $i = 1, 2, 3, 4$ are four distinct points on the circle $x^2 + y^2 = a^2$, then value of $m_1 + m_2 + m_3 + m_4$ equals

(a) 0 (b) –1 (c) 1 (d) $-a^2$

5. If $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$ and $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is

(a) – 1 (b) $\sqrt{10} + \sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

6. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ equals

(a) 2 (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) 1

7. Let in a triangle, any two sides and included angle are respectively $\sqrt{3} - 1$, $\sqrt{3} + 1$, 60°, then which of the following is not true?

(a) Triangle is obtuse

(b) One of the angle is 150°

(c) Third side is $\sqrt{6}$

(d) None of these

8. If ω is cube root of unity, then

$$\tan\left\{\left(\omega^{200} + \frac{1}{\omega^{200}}\right)\pi + \frac{\pi}{4}\right\} \text{ equals}$$

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) None of these

9. The equation of the straight line which passes through the point (–5, 4) and is such that the portion of it between the axes is divided by the given point in the ratio 1 : 2, is

(a) $\frac{x}{12} + \frac{y}{-\frac{15}{2}} = 1$ (b) $\frac{x}{-\frac{15}{2}} + \frac{y}{12} = 1$

(c) $\frac{x}{12} + \frac{y}{\frac{15}{2}} = 1$ (d) None of these

10. Let $y'(x) + y(x)\, g'(x) = g(x)\, g'(x)$, $y(0) = 0$, $x \in R$ where $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$, then the value of $y(2)$ equals

(a) 3 (b) 2 (c) 1 (d) 0

SECTION-II

Numerical Answer Type

11. The radius of the circle $\vec{r}^2+\vec{r}\cdot(2\hat{i}-2\hat{j}-4\hat{k})-19=0$ $=\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})+8$ is________.

12. If the area bounded by the curves $y = x^2$, $y = -x^2$ & $y^2 = 4x - 3$ is $\frac{a}{b}$, where H.C.F. $(a, b) = 1$, then value of $(a^2 + b^2)$ equals ________.

13. Two numbers x and y are randomly selected from the set $A = \{1, 2, 3, 4, 5, ..., 25\}$ and the probability that $x^2 - y^2$ is divisible by 3 is λ, then the value of $\frac{75\lambda}{41}$ is________.

14. If $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}-e+\frac{ex}{2}}{ex^2}=\dfrac{m}{n}$, where m and n are relatively prime positive integers, then value of $3m - n$ is ________.

15. If L be the length of common tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ and the circle $x^2 + y^2 = 16$ intercepted by the coordinate axis, then $\frac{\sqrt{3}L}{2}$ is ________.

SOLUTIONS

1. (b) : $\because\ S_{(P)}=1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{P}\ (P\in N)$

$\therefore\ S_{(1)} = 1$

$S_{(2)}=1+\frac{1}{2};\ S_{(3)}=1+\frac{1}{2}+\frac{1}{3};\ S_{(4)}=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

....

....

....

$S_{(P-1)}=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+............+\frac{1}{P-1}$

Adding the above relations vertically, we have

$$S_{(1)}+S_{(2)}+......+S_{(P-1)}=1\cdot(P-1)+\frac{1}{2}(P-2)+\frac{1}{3}(P-3)+..........+\frac{1}{P-1}$$

$$=1(P-1)+\frac{1}{2}(P-2)+\frac{1}{3}(P-3)+....+\frac{1}{(P-1)}(P-(P-1))$$

$$=P\left(1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{P-1}\right)$$

$-\ (1 + 1 + 1 + ...$ upto $(P - 1)$ times)

$= PS_{(P-1)} - (P-1) = PS_{(P-1)} - P + 1$

2. (b) : Here, repetition of things is allowed and at most r can be taken at a time.

Now, total number of things = n

Number of ways of taking 1 at a time = ${}^nP_1 = {}^nC_1 = n$

Number of ways of taking 2 at a time = n^2

Number of ways of taking 3 at a time = n^3

Continuing the process, to take r things at a time, the number of ways = n^r

$\therefore$ Total no. of ways of taking 1, 2, ..., r things at a time

$$=n+n^2+n^3+...+n^r=n(1+n+n^2+...+n^{r-1})=\frac{n(n^r-1)}{(n-1)}$$

3. (c) : Given $f(n)$ = square of the sum of the digits of n

$f^2(n)=f(f(n)), f^3(n)=f(f(f(n)))$ and so on

Now $f(2021) = (2 + 0 + 2 + 1)^2 = 25$

$\therefore\ f^2(2021) = f(25) = 49,$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

							1												7	
3	7		6	9	5		8					4	7		8	3	1		2	
		4				6								5				4		
	2		9		3		7						4		3		6		1	
	5			1			9						8			5			6	
	4		7		2		5						3		7		8		4	
		9				3								4				2		
	3		4	8	9		2	1					6		5	4	2		9	7
	1									3			5							
									8		4									
								2				5								
									2		9									
							3			2									6	
3	8		9	5	7		1					9	8		5	1	4		7	
		4				6								2				1		
	3		8		5		4						3		2		7		4	
	1			9			7						5			4			9	
	2		7		3		9						2		1		6		8	
		7				4								3				7		
	4		5	3	8		6	1					9		6	3	5		1	8
	6												6							

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$f^3(2021) = f(49) = (4+9)^2 = 169$$
$$f^4(2021) = f(169) = (1+6+9)^2 = 256,$$
$$f^5(2021) = f(256) = (2+5+6)^2 = 169$$

It is periodic function where $f^{2n}(2021)$ $= 256$ & $f^{2n+1}(2021) = 169\ \forall\ n \geq 2$

$$\frac{f^{2021}(2021) - f^{2020}(2021)}{f^{2023}(2021) - f^{2022}(2021)} = \frac{169-256}{169-256} = 1$$

4. (a) : Equation of circle is $x^2 + y^2 = a^2$

As $\left(m_i, \frac{1}{m_i}\right)$ lie on the circle

$$\therefore\ m_i^2 + \frac{1}{m_i^2} = a^2$$

$\Rightarrow\ m_i^4 - a^2 m_i^2 + 1 = 0$, which is fourth degree equation in m_i and have four roots say m_1, m_2, m_3, m_4

$$\therefore\ m_1 + m_2 + m_3 + m_4 = -\frac{\text{Coefficient of } m_i^3}{\text{Coefficient of } m_i^4} = \frac{0}{1} = 0$$

5. (c) : $[\vec{U}\ \vec{V}\ \vec{W}] = \vec{U} \cdot (\vec{V} \times \vec{W})$

$= |\vec{U}|\, |\vec{V} \times \vec{W}| \cos\phi$, ϕ is the angle between $\vec{U}$ and $\vec{V} \times \vec{W}$

Now, $|\vec{V} \times \vec{W}| \cos\phi \leq |\vec{V} \times \vec{W}|$

$\therefore$ Max. value of $[\vec{U}\,\vec{V}\,\vec{W}] = |\vec{V} \times \vec{W}| = \sqrt{59}$

$$\left[\because\ \vec{V} \times \vec{W} = 3\hat{i} - 7\hat{j} - \hat{k}\right]$$

6. (c) : Four diagonals are OP, AA', BB', CC'.

D.C's of OP are $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

D.C's of AA' are $< -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

D.C's of BB' are $< \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

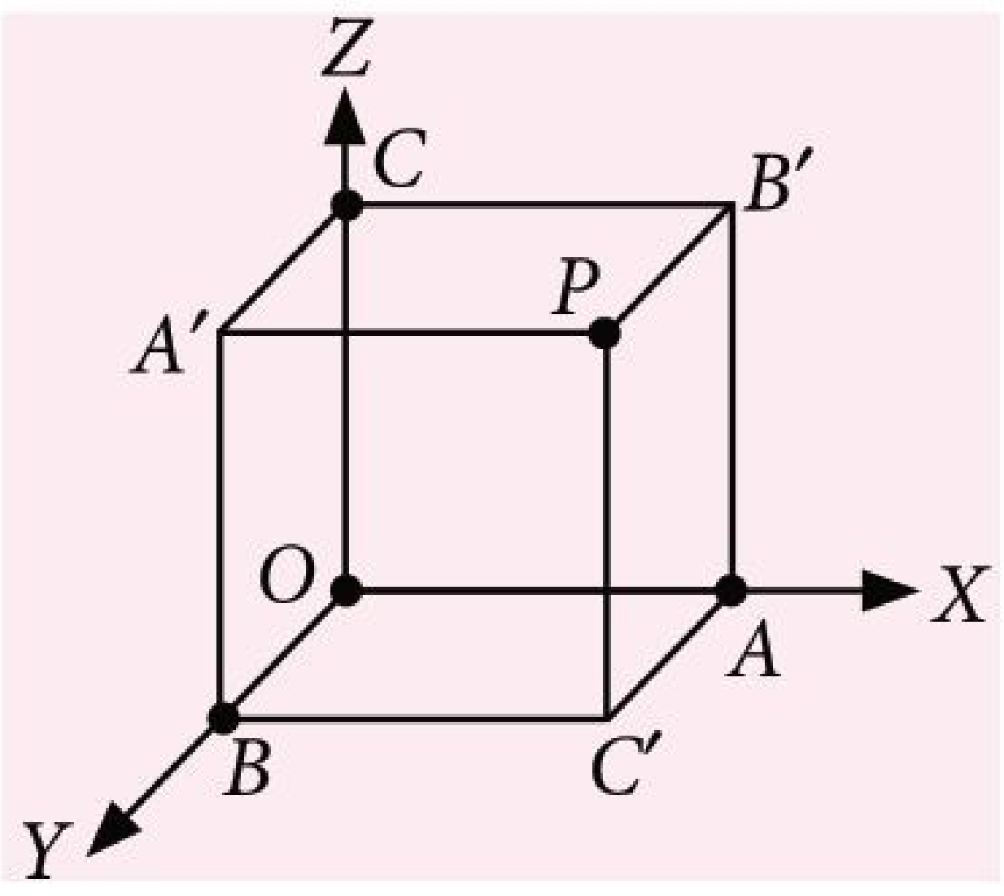
D.C's of CC' are $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} >$

Let l, m, n be the D.C's of line which makes the angle α, β, γ, δ with the four diagonals.

$$\therefore\ \cos\alpha = \frac{l+m+n}{\sqrt{3}},$$

$$\therefore\ \cos\beta = \frac{-l+m+n}{\sqrt{3}}$$

$$\therefore\ \cos\gamma = \frac{l-m+n}{\sqrt{3}}$$



$$\therefore\ \cos\delta = \frac{l+m-n}{\sqrt{3}}$$

$\therefore\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 [1+1+1+1] = \frac{4}{3}$$

$$\therefore\ \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 4 - \frac{4}{3} = \frac{8}{3}$$

7. (b) : Let two sides be b and c.

$\therefore\ \angle A = 60° \Rightarrow \angle B + \angle C = 120°$ ...(i)

Using tangent law, $\frac{b-c}{b+c}\cot\frac{A}{2} = \tan\frac{B-C}{2}$, we get

$\Rightarrow\ \angle B - \angle C = 90°$ ...(ii)

By (i) and (ii), $\angle B = 105°$, $\angle C = 15°$

So, the triangle is obtuse.

$$\text{Again, } a = \frac{b\sin A}{\sin B} = \left(\sqrt{3}+1\right)\frac{\sin 60°}{\sin 105°}$$

$$\Rightarrow a = \frac{\sqrt{2}\left(\sqrt{3}+1\right)}{\sqrt{3}+1} \times \sqrt{3} \ \Rightarrow\ a = \sqrt{6}$$

8. (a) : We have, $\tan\left\{\left(\omega^{200} + \frac{1}{\omega^{200}}\right)\pi + \frac{\pi}{4}\right\}$

$$= \tan\left\{(\omega^2 + \omega)\pi + \frac{\pi}{4}\right\} = \tan\left\{-\pi + \frac{\pi}{4}\right\}\ (\because 1 + \omega + \omega^2 = 0)$$

$$= \tan\left\{-\frac{3\pi}{4}\right\} = 1$$

9. (b)

10. (d) : We have, $y'(x) + y(x)g'(x) = g(x)g'(x)$

$\therefore\ \text{I.F.} = e^{\int g'(x)dx} = e^{g(x)}$

$\therefore$ Solution of differential equation is

$$y(x)e^{g(x)} = \int g(x)g'(x)e^{g(x)}dx$$

$$= g(x)\int g'(x)e^{g(x)}dx - \int\left(\frac{d}{dx}(g(x))\int g'(x)e^{g(x)}dx\right)dx$$

$$= g(x)e^{g(x)} - \int g'(x)e^{g(x)}dx = g(x)\,e^{g(x)} - e^{g(x)} + k$$

$\therefore\ y(x) = (g(x) - 1) + ke^{-g(x)}$

Now, $g(0) = 0 = y(0) \Rightarrow ke^0 = 1 \Rightarrow k = 1$

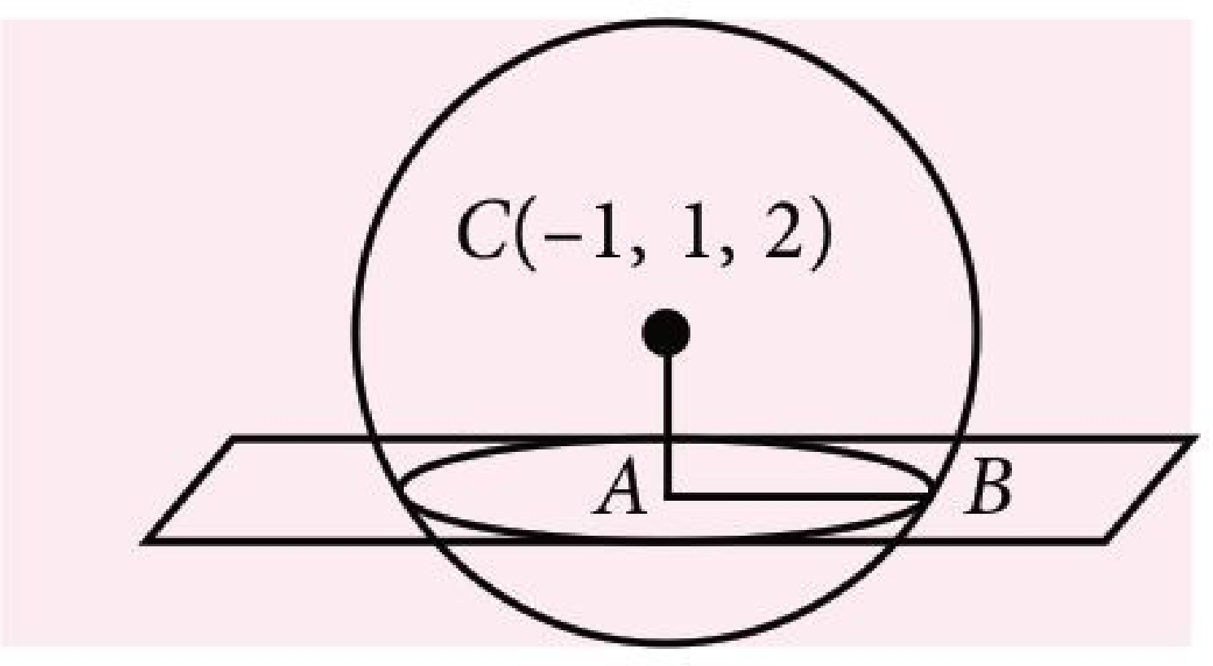
Again $y(x) = (g(x) - 1) + e^{-g(x)}$

$\therefore\ y(2) = (g(2) - 1) + e^{-g(2)}$

$y(2) = (0 - 1) + e^0 = -1 + 1 = 0$ $(\because\ g(2) = 0)$

11. (4) : Equation of sphere is $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ and plane is $x - 2y + 2z + 8 = 0$

Now, centre of sphere is $(-1, 1, 2)$



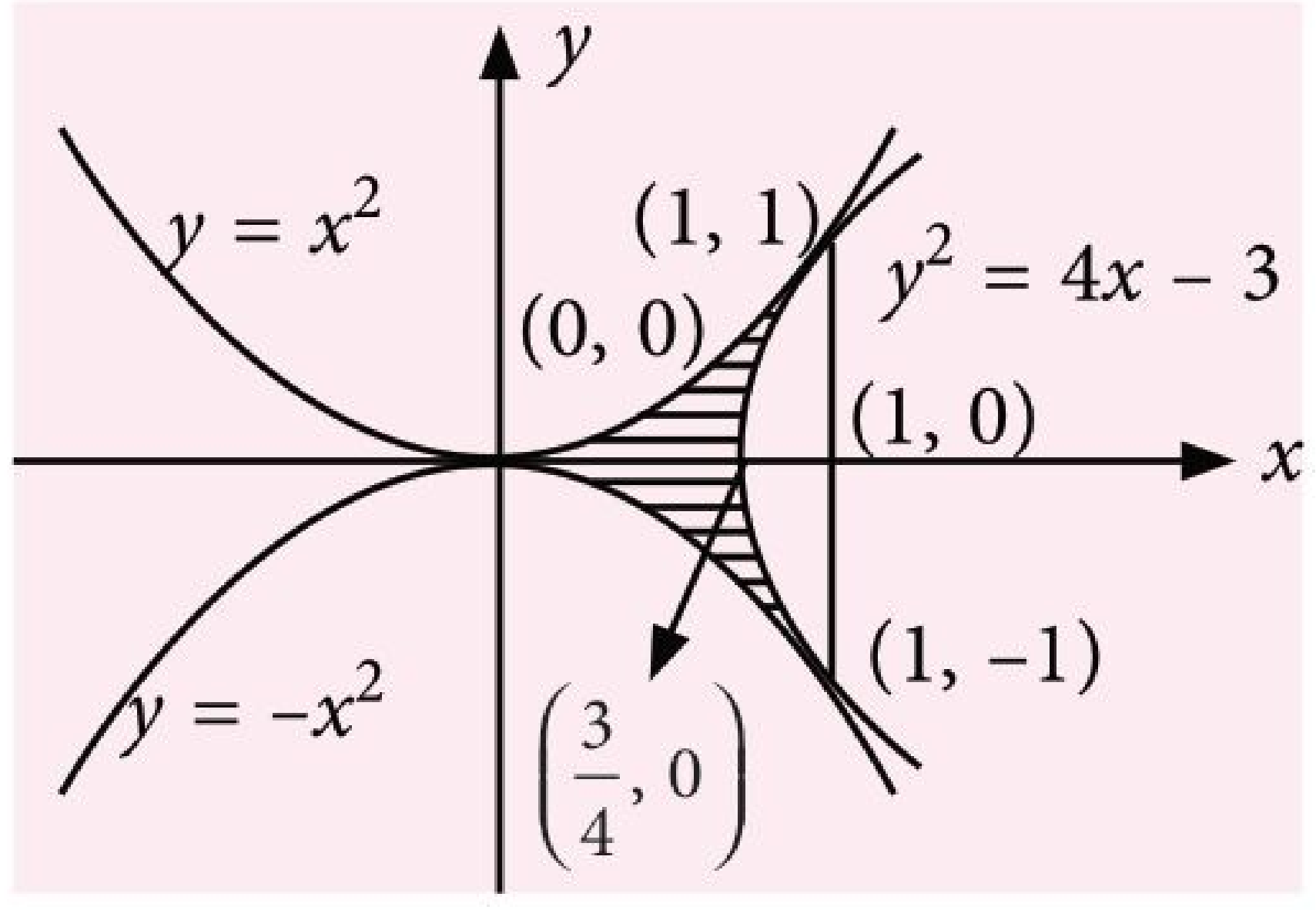
$R = CB = \sqrt{1+1+4+19} = 5$

$CA = \dfrac{|-1-2+4+8|}{\sqrt{4+4+1}} = 3$

Now, required radius, $AB = \sqrt{(CB)^2 - (CA)^2}$

$= \sqrt{25-9} = 4$

12. (10) : Point of intersection of $y = x^2$ & $y^2 = 4x - 3$ is (1, 1). Point of intersection of $y = -x^2$ & $y^2 = 4x - 3$ is (1, –1). Point of intersection of $y = x^2$ & $y = -x^2$ is (0, 0).



$\therefore$ Required area $= \dfrac{a}{b}$

$$= 2\left[\int_0^1 x^2 dx - \int_{3/4}^1 \sqrt{4x-3}\, dx\right] = 2\left[\frac{1}{3} - \frac{1}{6}\right] = \frac{1}{3}$$

$\therefore a^2 + b^2 = 10$

13. (1) : Here, $A = \{1, 2, 3, 4, 5, ..., 25\}$

Let P = Subset of A in which each number is multiple of 3

Q = Subset of A in which numbers are not multiple of 3

$\therefore$ $P = \{3, 6, 9, 12, 15, 18, 21, 24\}$. $\therefore$ $n(P) = 8$ and

$Q = A - x = \{1, 2, 4, 5, 7, ..., 25\}$. $\therefore$ $n(Q) = 17$

Now, the number $x^2 - y^2$ will be divisible by 3 if both numbers are selected either from P or Q.

$\therefore$ Number of favourable cases $= {}^8C_2 + {}^{17}C_2 = 164$

and total cases $= {}^{25}C_2 = 300$

$\therefore$ Required probability $= \dfrac{\text{favourable cases}}{\text{total cases}} = \dfrac{164}{300}$

$\therefore$ $\lambda = \dfrac{164}{300} = \dfrac{41}{75} \Rightarrow \dfrac{75\lambda}{41} = 1$

14. (9) : Let $L = \lim\limits_{x\to 0} \dfrac{e \cdot e^{-\frac{x}{2}+\frac{x^2}{3}\cdots} - e + \frac{ex}{2}}{ex^2}$

$$\left[\because\ (1+x)^{1/x} = e^{\frac{1}{x}\log(1+x)} = e^{\left(1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\ldots\ldots\right)}\right]$$

$$= \lim_{x\to 0} \frac{e^{-\frac{x}{2}+\frac{x^2}{3}-\ldots\ldots} - 1 + \frac{x}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{1+\left(-\frac{x}{2}+\frac{x^2}{3}-\ldots\right)+\frac{\left(-\frac{x}{2}+\frac{x^2}{3}-\ldots\right)^2}{2}\ldots-1+\frac{x}{2}}{x^2}$$

$= \dfrac{1}{3} + \dfrac{1}{8} = \dfrac{11}{24} = \dfrac{m}{n}$

$\therefore$ $3m - n = 9$

15. (7) : The equation of the tangent at $(5\cos\theta, 2\sin\theta)$ to the ellipse is $\dfrac{x}{5}\cos\theta + \dfrac{y}{2}\sin\theta = 1$.

If it is a tangent to the circle, then $\dfrac{1}{\sqrt{\dfrac{\cos^2\theta}{25} + \dfrac{\sin^2\theta}{4}}} = 4$

$\Rightarrow$ $\cos\theta = \dfrac{10}{4\sqrt{7}}$, $\sin\theta = \dfrac{\sqrt{3}}{2\sqrt{7}}$

Let A and B be the points where the tangent meets the coordinate axes, then

$A\left(\dfrac{5}{\cos\theta}, 0\right)$, $B\left(0, \dfrac{2}{\sin\theta}\right)$; $L = \sqrt{\dfrac{25}{\cos^2\theta} + \dfrac{4}{\sin^2\theta}} = \dfrac{14}{\sqrt{3}}$

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OLYMPIAD CORNER

1. Let a, b, c be real numbers. Their pairwise sum $a + b$, $b + c$ and $c + a$ are written on three round cards and their pairwise products ab, bc and ca are written on three square cards. We call (a, b, c) a tadpole if we can form three pairs of cards, each consisting of one round card and one square card with the same number on both. An example of a tadpole is $(0, 0, 0)$.

(a) Find all possible tadpoles of the form (a, a, a).

(b) Prove that there is a tadpole that is not of the form (a, a, a). (you do not have to find the actual values of a, b and c.)

2. Prove that for all positive real numbers a, b, c the following inequality holds:

$$2\sqrt{bc+ca+ab} \le \sqrt{3}\sqrt[3]{(b+c)(c+a)(a+b)}.$$

3. For natural $n \ge 1$, let $s(n)$ denotes the sum of all positive divisors of n. Prove that for every integer $n > 1$ the product $s(n - 1)s(n)s(n + 1)$ is even.

4. Given a circle k of centre M and radius r, let AB be a fixed diameter of k and let K be a fixed point of segment AM. Denote by t the line tangent to k at A. For any chord CD (other than AB) passing through K construct P and Q as the points of intersection of lines BC and BD with t. Prove that the product $AP \cdot AQ$ remains constant as the chord CD varies.

5. There are n black marbles and two red marbles in a jar. One by one, marbles are drawn at random out of the jar. Jeanette wins as soon as two black marbles are drawn, and Fraserette wins as soon as two red marbles are drawn. The game continues until one of the two wins. Let $J(n)$ and $F(n)$ be the two probabilities that Jeanette and Fraserette win, respectively.

(a) Determine the value of $F(1) + F(2) + \ldots + F(3992)$.

(b) As n approaches infinity, what does $J(2) \times J(3) \times J(4) \ldots \times J(n)$ approach?

6. There are n houses situated around a certain lake, in a circle. Each is painted one of k colours, chosen at random. Find the probability that no two neighbouring houses are of the same colour.

7. $BCLK$ is the square constructed outwardly on side BC of an acute triangle ABC. Let CD be the altitude of ΔABC (with D on AB), and let H be the orthocentre of ΔABC. If the lines AK and CD meet at P, then show that $\frac{HP}{PD} = \frac{AB}{CD}$.

SOLUTIONS

1. (a) If $b = c = a$, then there are 3 pairs of cards $2a$ and a^2. Now, $a^2 = 2a$ iff $a = 0$ or $a = 2$. Thus, there are exactly two tadpoles of the form (a, a, a), namely $(0, 0, 0)$ and $(2, 2, 2)$.

(b) **Case I.** Two of a, b, c are equal: say a, a, b $(b \neq a)$. This leads to pairings

(i) $(2a, a^2)$, $(a + b, ab)$ and $(a + b, ab)$.

The first pair of which implies $a = 0$ or 2

$a = 0 \Rightarrow ab = 0 \Rightarrow a + b = 0 \Rightarrow b = 0 = a$,

$a = 2 \Rightarrow b + 2 = 2b \Rightarrow b = 2 = a$,

(ii) $(2a, ab)$, $(a + b, a^2)$ and $(a + b, ab)$. Here, the last two pairs give $a^2 = ab$ implying $a = 0$ or $a = b$.

Now, $a = 0 \Rightarrow b = 0 = a$.

Case II. a, b, c are all distinct. [Throughout, the assumption is made implicitly that a, b, c do not take values which would render any of the others unbounded].

(i) $(a + b, ab)$, $(b + c, bc)$ and $(c + a, ca)$.

$ab = a + b \Rightarrow (a - 1)(b - 1) = 1$ and $b = 1 + \frac{1}{a-1}$.

Similarly

$$c = \frac{1}{1+\frac{1}{b-1}} \text{ and } a = 1 + \frac{1}{c-1} = 1 + \frac{1}{1+\frac{1}{b-1}} = b.$$

(ii) $(a + b, bc)$, $(b + c, ab)$, $(c + a, ac)$.

Now $a + b = bc \Rightarrow c = 1 + 1/b$. Then $b + 1 + a/b = ab$ $\Rightarrow b + 1 = a(b - 1/b) \Rightarrow a = b/(b - 1)$ or $b = -1$.

$$a = \frac{b}{b-1} \Rightarrow c = 1 + \frac{1}{b-1} = \frac{b}{b-1} = a.$$

$b = -1 \Rightarrow a^2 - a + 1 = 0$ and a is not real.

(iii) $(a + b, ca)$, $(c + a, ab)$ and $(c + a, bc)$.

Now $\left.\begin{array}{l} a+b=ca \Rightarrow b=a(c-1) \\ b+c=ab \Rightarrow b=\dfrac{c}{a-1} \end{array}\right\} \Rightarrow a(a-1)=\dfrac{c}{c-1}.$

Also $c + a = ac(c - 1) \Rightarrow a=\dfrac{c}{c^2-c-1}$.

so that $\left(\dfrac{c}{c^2-c-1}\right)\left(\dfrac{2c-c^2+1}{c^2-c-1}\right)=\dfrac{c}{c-1}$.

This gives $c^4 - c^3 - 4c^2 + 3c + 2 = 0$

$\Rightarrow\ (c - 2)(c^3 + c^2 - 2c - 1) = 0.$

Now $c = 2$ gives $a^2 - a - 2 = 0 \Rightarrow a = 2$, a contradiction, or $a = -1 \Rightarrow b = -1$, a contradiction.

Thus $c^3 + c^2 - 2c + 1 = 0$ is required to have a real solution, which it must do since complex roots occur in conjugate pairs. (Note only (iii) is necesssary in order to establish the existence of solutions, provided it is now checked that $c - 1 \neq 0$ and $c^2 - c - 1 \neq 0$. The other cases are included for completeness.)

In fact, using Cardan's method and complex numbers, the three real roots of the cubic $c^3 + c^2 - 2c - 1 = 0$ can be found. They are

$$c_1=\frac{2\sqrt{7}}{3}\cos\frac{\theta}{3}-\frac{1}{3},\quad c_2=\frac{2\sqrt{7}}{3}\cos\left(\frac{2\pi-\theta}{3}\right)-\frac{1}{3}$$

and $c_3=\dfrac{2\sqrt{7}}{3}\cos\left(\dfrac{2\pi+\theta}{3}\right)-\dfrac{1}{3}$, where $\theta=\cos^{-1}\left(\dfrac{1}{\sqrt{28}}\right)$.

There are then 6 tadpoles not of the form (a, a, a) being the six permutations of (c_1, c_2, c_3).

2. It is enough to prove that for all positive real numbers a, b, c the following inequality holds:

$$64(bc + ca + ab)^3 \leq 27(a + b)^2(a + c)^2(b + c)^2$$

or $\ 64 \cdot 3(bc + ca + ab)(bc + ca + ab)^2 \leq 81[(a + b)(a + c)(b + c)]^2.$

It is known that $3(bc + ca + ab) \leq (a + b + c)^2$. Thus, it is enough to prove one of the following equivalent inequalities:

$8(a + b + c)(bc + ca + ab) \leq 9(a + b)(a + c)(b + c)$

$\Rightarrow\ 8(a + b)(a + c)(b + c) + 8abc \leq 9(a + b)(a + c)(b + c)$

$\Rightarrow\ 8abc \leq (a + b)(a + c)(b + c),$

which is well known to hold.

3. We show equivalently that at least one of $s(n - 1)$, $s(n)$ and $s(n + 1)$ must be even. Let $n = P_1^{t_1} P_2^{t_2} \ldots P_k^{t_k}$ be the prime powers decomposition of n, where the P_i's are distinct primes. Then it is well known that

$$s(n)=\prod_{i=1}^{k}(1+P_i+P_i^2+\ldots+P_i^{t_i}).$$

If $P_i = 2$, then the factor $p_i = 1 + P_i^2 + \ldots + P_i^{t_i}$ is clearly odd. If P_i is odd, then clearly p_i is odd iff t_i is even. Hence $s(n)$ is odd if and only if $n = 2^d q^2$, where $d \geq 0$ and q is odd. Suppose, contrary to what we claim, that $s(n - 1)$, $s(n)$ and $s(n + 1)$ are all odd. Then $n - 1 = 2^a q_1^2$, $n = 2^b q_2^2$ and $n + 1 = 2^c q_3^2$ where $a, b, c \geq 0$ and $q_i (i = 1, 2, 3)$ are all odd. We consider two cases:

Case (i) : If n is even, then $a = c = 0$. Thus $n - 1 = q_1^2$ and $n + 1 = q_3^2$ which imply $(q_3 - q_1)(q_3 + q_1) = 2$ and thus $q_3 - q_1 = 1$ and $q_3 + q_1 = 2$, which clearly yields no integer solutions, a contradiction.

Case (ii). If n is odd, then $b = 0$, $a, c \geq 1$ and $n = q_2^2$. Thus

$$q_2^2 - 2^a q_1^2 = 1 \qquad \text{...(i)}$$

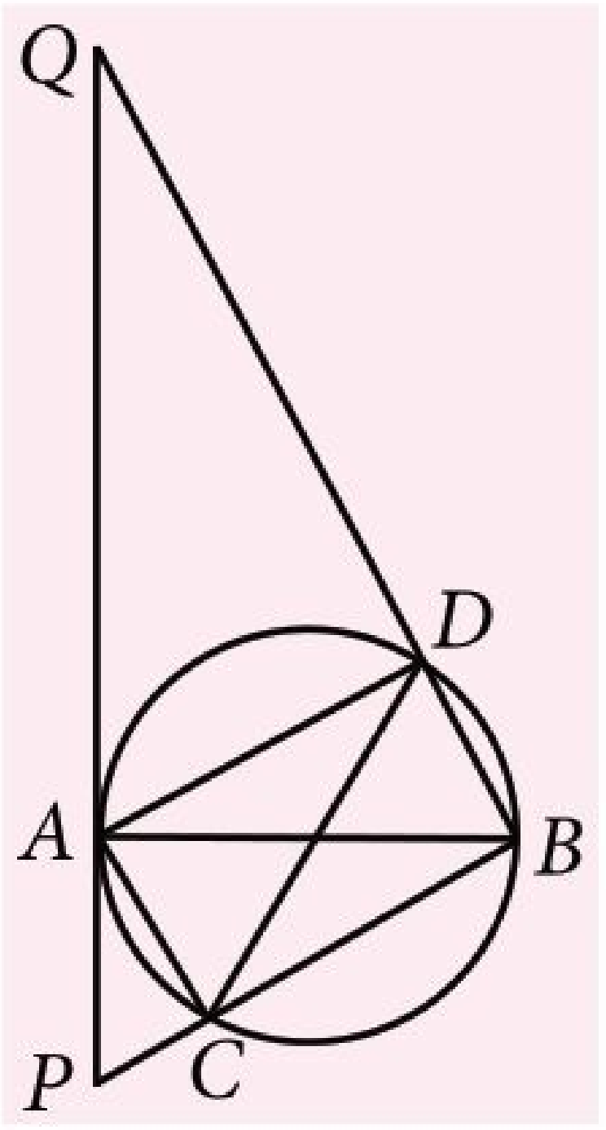
Furthermore $2^c q_3^2 = n + 1 = q_2^2 + 1$. If $c \geq 2$, then $2^c q_3^2 \equiv 0 \pmod 4$ while $q_2^2 + 1 \equiv 2 \pmod 4$. Since, the square of any odd integer is congruent to 1 (mod 4). Thus $c = 1$ and from $2q_3^2 = 2^1 a_1^2 + 2$ we get

$$q_3^2 - 2^{a-1} q_1^2 = 1 \qquad \text{...(ii)}$$

Note that equation $x^2 - y^2 = 1$ has no solutions in positive integers since $(x - y)(x + y) = 1$ is clearly impossible. Hence if a is even then (i) is not solvable while if a is odd then (ii) is not solvable. Again we have a contradiction.

4. Because $\angle BAP = 90°$ and $\angle ACB = 90°$ we get ΔAPB similar to ΔCAB. Thus we have $\dfrac{AP}{AB}=\dfrac{CA}{CB}$. Similarly we have $\dfrac{AQ}{AB}=\dfrac{DA}{DB}$. Multiplying both sides gives

$$\frac{AP\cdot AQ}{AB^2}=\frac{CA\cdot DA}{CB\cdot DB}. \qquad \text{...(i)}$$



As A, C, B, D are concyclic we get ΔACK similar to ΔDBK and ΔDAK similar to ΔBCK. Thus we have

$$\frac{KA}{BD}=\frac{CK}{BK} \text{ and } \frac{DA}{BC}=\frac{AK}{CK}.$$

Therefore

$$\frac{CA}{BD}\cdot\frac{DA}{BC}=\frac{CK}{BK}\cdot\frac{AK}{CK}=\frac{AK}{BK} \qquad \text{...(ii)}$$

From (i) and (ii) we get, $\dfrac{AP\cdot AQ}{AB^2}=\dfrac{AK}{BK}$.

Hence we have $AP \cdot AQ = AB^2 \times AK/BK$, a constant.

5. (a) If Fraserette wins, the balls must be drawn in one of the following three ways: red, red; red, black, red; or black, red, red. This must be the case, as otherwise two black balls will be drawn and Jeanette will win. Hence the probability that Fraserette wins is the sum of the probabilities of each of three cases above.

Thus, $F(n)=\dfrac{2}{n+2}\cdot\dfrac{1}{n+1}+\dfrac{2}{n+2}\cdot\dfrac{n}{n+1}\cdot\dfrac{1}{n}+\dfrac{n}{n+2}\cdot\dfrac{2}{n+1}\cdot\dfrac{1}{n}$

$$=\frac{2}{(n+1)(n+2)}+\frac{2}{(n+1)(n+2)}+\frac{2}{(n+1)(n+2)}$$

$$=\frac{6}{(n+1)(n+2)}.$$

Note that $F(n)=\frac{6}{n+1}-\frac{6}{n+2}$, so we can use a telescoping series to calculate the desired sum.

We have, $F(1)+F(2)+\ldots+F(3992)$

$$=\left(\frac{6}{2}-\frac{6}{3}\right)+\left(\frac{6}{3}-\frac{6}{4}\right)+\left(\frac{6}{4}-\frac{6}{5}\right)+\ldots+\left(\frac{6}{3993}-\frac{6}{3994}\right)$$

$$=\frac{6}{2}-\frac{6}{3994}=3-\frac{3}{1997}=\frac{5988}{1997}.$$

(b) Now $J(n)=1-F(n)$

$$=1-\frac{6}{(n+1)(n+2)}=\frac{n^2+3n-4}{(n+1)(n+2)}=\frac{(n-1)(n+4)}{(n+1)(n+2)}$$

Thus, $J(2)\times J(3)\times J(4)\times\ldots\times J(n)$

$$=\frac{1\cdot 6}{3\cdot 4}\cdot\frac{2\cdot 7}{4\cdot 5}\cdot\frac{3\cdot 8}{5\cdot 6}\cdot\frac{4\cdot 9}{6\cdot 7}\cdots\frac{(n-1)\cdot(n+4)}{(n+1)\cdot(n+2)}$$

$$=\frac{1\cdot 2\cdot 3\cdot 4\cdots(n-2)\cdot(n-1)}{3\cdot 4\cdot 5\cdot 6\cdots n\cdot(n+1)}\cdot\frac{6\cdot 7\cdot 8\cdot 9\cdot(n+3)\cdot(n+4)}{4\cdot 5\cdot 6\cdot 7\cdots(n+1)\cdot(n+2)}$$

$$=\frac{1\cdot 2}{n\cdot(n+1)}\cdot\frac{(n+3)\cdot(n+4)}{4\cdot 5}=\frac{1}{10}\cdot\frac{(n+3)(n+4)}{n(n+1)}$$

$$=\frac{1}{10}\cdot\frac{n+3}{n}\cdot\frac{n+4}{n+1}.$$

Thus, as n approaches infinity, $\frac{n+3}{n}=1+\frac{3}{n}$ and $\frac{n+4}{n+1}=1+\frac{3}{n+1}$ both approach 1, and so $J(2)\times J(3)\times J(4)\times\ldots\times J(n)$ approaches $\frac{1}{10}$.

6. Let $P(n, k)$ denotes the desired probability. Let $h(a)$ denotes the colour of the a^{th} house, starting from a fixed house, going around the circle.

Without loss of generality, we may fix $h(1)=1$. Under this condition, let $p(n, a, b)$ denote the probability that $h(a)=b$, if indeed no two adjacent houses are painted the same colour.

With $h(1)=1$, there are $k-1$ ways to colour the second house (out of k colours), the third house, etc., until the $(n-1)^{\text{th}}$ house. If $h(n-1)=1$, then there are $k-1$ ways to colour the n^{th} house. However, if $h(n-1)\neq 1$, then there are $k-2$ ways to colour the n^{th} house.

Hence, $$P(n,k)=\left(\frac{k-1}{k}\right)^{n-2}\left(\frac{k-1}{k}p(n, n-1, 1)+\frac{k-2}{k}(1-p(n, n-1, 1))\right).$$

But for all a, $p(n, a, 1)=\frac{1}{k-1}(1-p(n, a-1, 1))$

$$=\frac{1}{k-1}-\frac{1}{k-1}\,p(n, a-1, 1)$$

So that $p(n, a, 1)-\frac{1}{k}=\frac{1}{k(k-1)}-\frac{1}{k-1}\,p(n, a-1, 1)$

$$=\frac{-1}{k-1}\left(p(n, a-1, 1)-\frac{1}{k}\right)$$

$$=\left(\frac{-1}{k-1}\right)^2\left(p(n, a-2, 1)-\frac{1}{k}\right)$$

$$\vdots$$

$$=\left(\frac{-1}{k-1}\right)^{a-1}\left(p(n, 1, 1)-\frac{1}{k}\right)$$

$$=\frac{(-1)^{a-1}}{(k-1)^{a-1}}\cdot\frac{k-1}{k}=\frac{(-1)^{a-1}}{k(k-1)^{a-2}}.$$

Therefore, $p(n, n-1, 1)=\frac{(-1)^n}{k(k-1)^{n-3}}$,

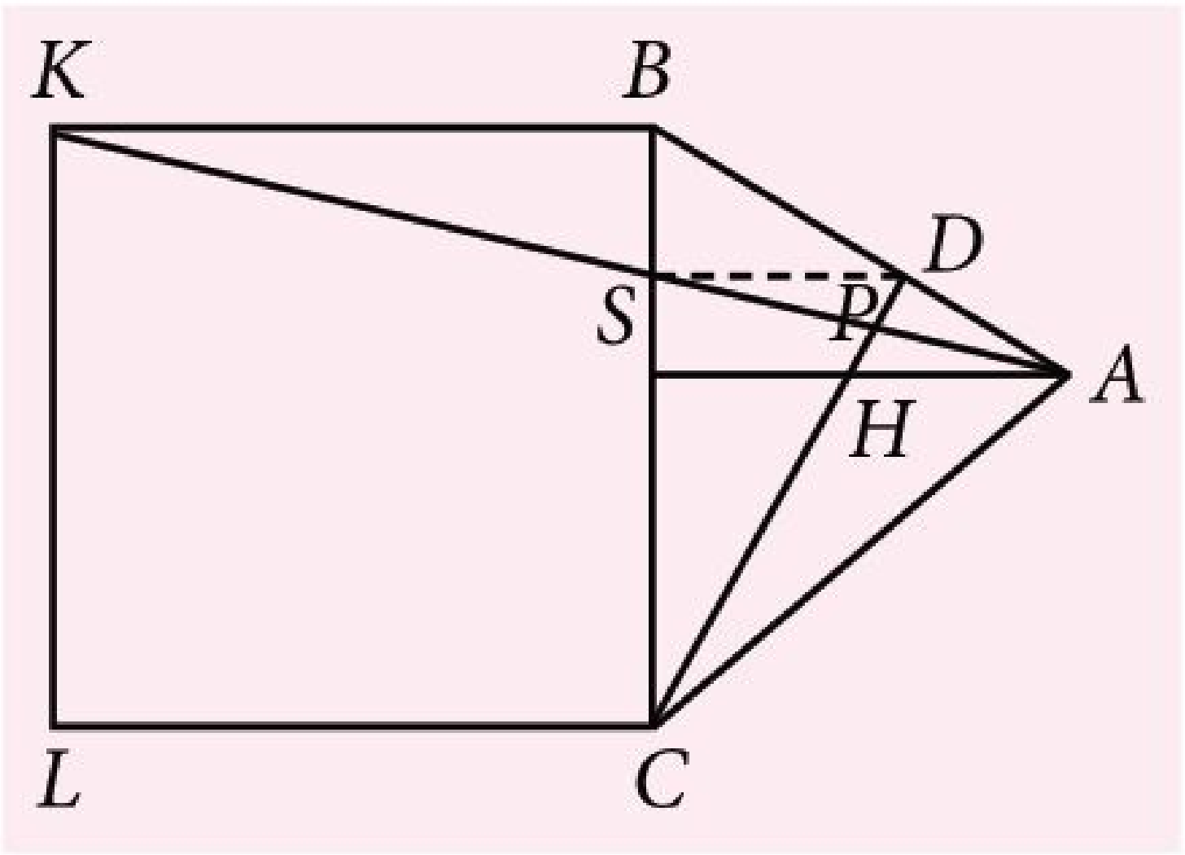
and $$p(n,k)=\left(\frac{k-1}{k}\right)^{n-2}\left[\frac{k-1}{k}\left(\frac{1}{k}+\frac{(-1)^n}{k(k-1)^{n-3}}\right)+\frac{k-2}{k}\left(1-\frac{1}{k}-\frac{(-1)^n}{k(k-1)^{n-3}}\right)\right]$$

$$=\left(\frac{k-1}{k}\right)^{n-2}\left(\frac{k-1}{k^2}+\frac{(-1)^n(k-1)}{k^2(k-1)^{n-3}}+\frac{(k-1)(k-2)}{k^2}-\frac{(-1)^n(k-2)}{k^2(k-1)^{n-3}}\right)$$

$$=\left(\frac{k-1}{k}\right)^{n-2}\left(\frac{(k-1)^2}{k^2}+\frac{(-1)^n}{k^2(k-1)^{n-3}}\right)$$

$$=\left(\frac{k-1}{k}\right)^{n}+\frac{(-1)^n(k-1)}{k^n}.$$

7. Let S be a point on AK such that $DS\perp BC$ and so $DS\parallel AH\parallel BK$.



Since $AH\parallel DS$, we get

$$\frac{HP}{PD}=\frac{AH}{DS}\qquad\ldots\text{(i)}$$

Since $DS\parallel BK$ and $BK=BC$, we have

$$\frac{AD}{AB}=\frac{DS}{BK}=\frac{DS}{BC}\qquad\ldots\text{(ii)}$$

Since $AH\perp BC$ and $CD\perp AB$ we get $\angle HAD=\angle BCD$. Moreover we have $\angle HDA=\angle BDC\ (=90°)$, so that $\Delta HAD\sim\angle BCD$. Thus

$$\frac{AH}{BC}=\frac{AD}{CD}\qquad\ldots\text{(iii)}$$

From (ii) and (iii) we have

$\frac{AH}{DS}=\frac{AH}{BC}\cdot\frac{BC}{DS}=\frac{AD}{CD}\cdot\frac{AB}{AD}=\frac{AB}{CD}$, so that we obtain from (i) that $\frac{HP}{PD}=\frac{AB}{CD}$

◆◆

Y?U ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Find the values of a, b, c if the following limit exists :

$$\lim_{x\to 0}\frac{ae^x - b\ln(1+x) + cxe^{-x}}{x^2\sin x} = 2$$ ***(Umang, Delhi)***

Ans. Let $L = \lim_{x\to 0}\frac{ae^x - b\ln(1+x) + cxe^{-x}}{x^2\sin x}$

$$= \lim_{x\to 0}\frac{ae^x + cxe^{-x} - b\ln(1+x)}{x^3}\cdot\frac{x}{\sin x}$$

$$= \lim_{x\to 0}\frac{ae^x + cxe^{-x} - b\ln(1+x)}{x^3}$$

$$= \lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\ldots\right)+cx\left(1-x+\frac{x^2}{2!}-\ldots\right) -b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)}{x^3}$$

$$= \lim_{x\to 0}\frac{a+(a+c-b)x+(a-2c+b)\frac{x^2}{2}+\left(\frac{a}{3!}+\frac{c}{2!}-\frac{b}{3}\right)x^3+\ldots}{x^3}$$

For a finite non-zero limit to exist, we have

$a = 0$ ...(i) $a + c - b = 0$ ...(ii)

$a - 2c + b = 0$ ...(iii)

and $\frac{a}{6}+\frac{c}{2}-\frac{b}{3} = 2$ [according to the given condition]

i.e. $a + 3c - 2b = 12$ ...(iv)

Solving (i), (ii), (iii) and (iv), we can see that there is no value of a, b, c which satisfies all the equations.

2. Solve the equation $4^{x+1.5} + 9^{x+0.5} = 10\cdot 6^x$.

(Anjali, Rajasthan)

Ans. The given equation can be written as,

$4^x\cdot 4^{3/2} + 9^x. 9^{1/2} = 10\cdot 6^x$

$$\Rightarrow\ 8\cdot 4^x + 3\cdot 9^x = 10\cdot 6^x \Rightarrow 8\cdot\left(\frac{2}{3}\right)^x + 3\cdot\left(\frac{3}{2}\right)^x = 10$$

[Dividing throughout by 6^x]

$$\Rightarrow\ 8y+\frac{3}{y} = 10 \qquad \text{[Putting } (2/3)^x = y]$$

$$\Rightarrow\ 8y^2 - 10y + 3 = 0$$

$$\Rightarrow\ y = \frac{10\pm\sqrt{4}}{16} = \frac{3}{4},\frac{1}{2} \Rightarrow \left(\frac{2}{3}\right)^x = \frac{3}{4},\frac{1}{2} \Rightarrow \left(\frac{3}{2}\right)^x = \frac{4}{3}, 2$$

$$\Rightarrow\ x\log\left(\frac{3}{2}\right) = \log\left(\frac{4}{3}\right), \log 2$$

$$\Rightarrow\ x = \frac{\log\left(\frac{4}{3}\right)}{\log\left(\frac{3}{2}\right)}, \frac{\log 2}{\log\left(\frac{3}{2}\right)} = \log_{1.5}\left(\frac{4}{3}\right), \log_{1.5} 2$$

3. Consider the function $f(x) = \int_0^x (1+t^3)^{-1/2}\,dt$. If g is the inverse of f, then find the value of $\frac{g''}{g^2}$.

(Prachi Aggarwal, Maharashtra)

Ans. We have, $f(x) = \int_0^x (1+t^3)^{-1/2}\,dt$

Now, $f\{g(x)\} = \int_0^{g(x)} (1+t^3)^{-1/2}\,dt$

$$\Rightarrow\ x = \int_0^{g(x)} (1+t^3)^{-1/2}\,dt \qquad \text{...(i)}$$

[$\because$ g is inverse of $f \Rightarrow f\{g(x)\} = x$]

Differentiating both sides of (i) w.r.t. x, we get

$1 = (1+g^3)^{-1/2}\cdot g' \ \Rightarrow\ (g')^2 = 1 + g^3$ ...(ii)

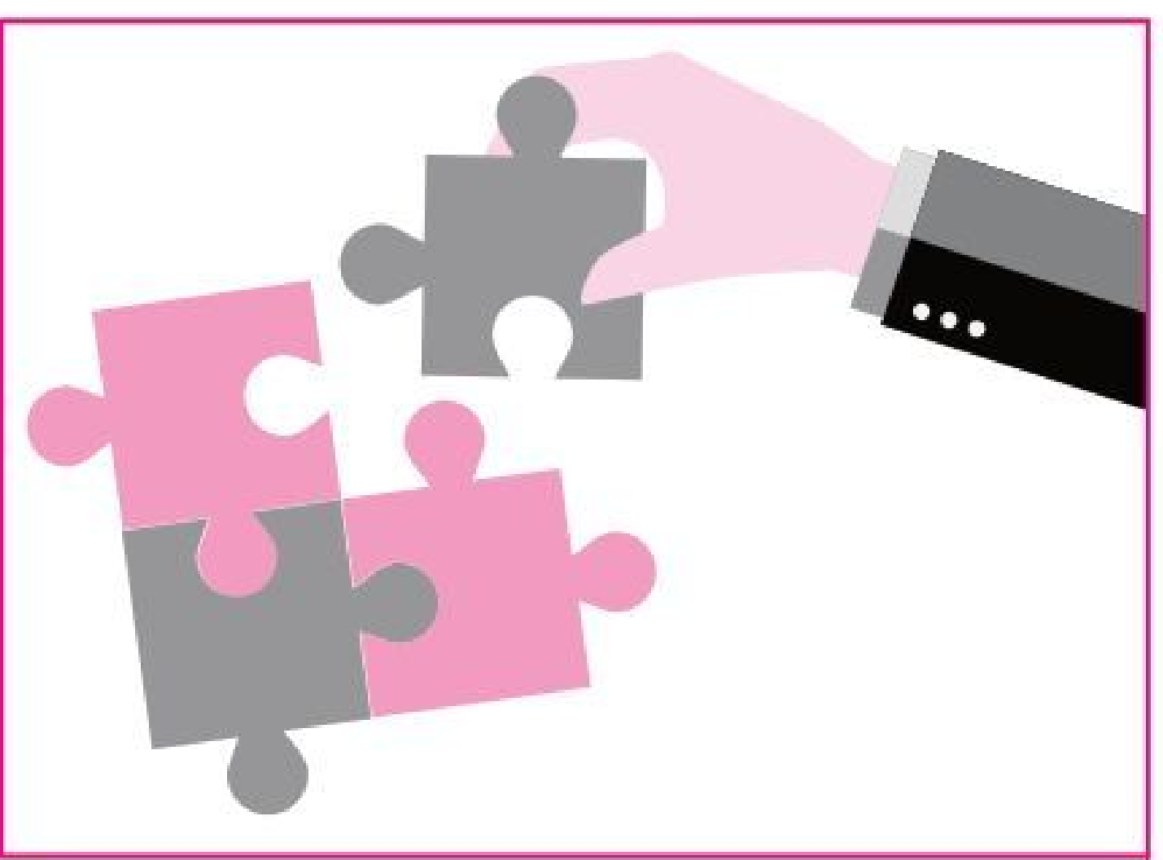
Again, differentiating both sides of (ii) w.r.t. x, we get

$2g'g'' = 3g^2g'$

Thus, $\frac{g''}{g^2} = \frac{3}{2}$.

◆ ◆

PUZZLE CORNER



SOLUTION · OCTOBER 2021

6+ 3	2	18× 6	1	4 4	3− 5
2÷ 4	1	3	1− 6	5	2
2	11+ 6	4− 5	1− 3	3− 1	4
5− 6	5	1	4	5+ 2	3
1	12+ 4	30× 2	5	3	6 6
5	3	2− 4	2	5− 6	1

CBSE warm-up!

CLASS-XI

TERM-I OBJECTIVE TYPE QUESTIONS

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

SECTION-A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1. Find the multiplicative inverse of $\frac{3+4i}{4-5i}$.

(a) $\frac{8}{25}+i\frac{31}{25}$ (b) $\frac{-8}{25}+i\frac{31}{25}$

(c) $\frac{-8}{25}-i\frac{31}{25}$ (d) $\frac{8}{25}-i\frac{31}{25}$

2. The line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ and $(x, 24)$. Find the value of x.

(a) 1 (b) 2

(c) 3 (d) 4

3. The real values of x and y for which the following equality hold, are respectively

$(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$

(a) 2, 3 or –2, 1/3 (b) 1, 3 or –1, 1/3

(c) 2, 1/3 or –2, 3 (d) 2, 1/3 or –2, –1/3

4. If the sum of n terms of an A.P. is $nP+\frac{1}{2}n(n-1)Q$, where P and Q are constants, then find the common difference.

(a) $P + Q$ (b) P (c) $P - Q$ (d) Q

5. The tabular form set $A = \{x : x \text{ is an odd integer and } 3 \le x \le 13\}$ is

(a) {3, 7, 9, 11, 13} (b) {3, 5, 9, 13}

(c) {3, 5, 7, 9, 11, 13} (d) None of these

6. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7. Then $p^2 + q^2 =$

(a) 42 (b) 49 (c) 50 (d) 51

7. Find the value of $\lim\limits_{x\to 1}\frac{x^{1/3}-1}{x^{1/6}-1}$.

(a) 2 (b) 1 (c) 0 (d) –2

8. The complex number $i^9 + i^{19}$ in the form of $a + ib$ is

(a) $2 + i3$ (b) $1 + i$ (c) $0 + i0$ (d) $2 + i$

9. The number of subsets of a set having 'n' elements is
(a) 2^{n^2} (b) 2^n
(c) 2^{n+1} (d) 2^{n+2}

10. Solve $x^2 + x + 1 = 0$.
(a) $\pm i$ (b) $\pm(1/2)i$
(c) $\frac{-1 \pm \sqrt{3}i}{2}$ (d) $\frac{\pm\sqrt{3}i}{2}$

11. The relation R defined on set $A = \{x : |x| < 3, x \in I\}$ by $R = \{(x, y) : y = |x|\}$ is
(a) $\{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$
(b) $\{(-2, -2), (-2, 2), (-1, 1), (0, 0), (1, -2), (1, 2), (2, -1), (2, -2)\}$
(c) $\{(0, 0), (1, 1), (2, 2)\}$
(d) None of these

12. The number of terms in the A.P. 3, 7, 11, ..., 407 is
(a) 102 (b) 101 (c) 100 (d) 104

13. $\lim\limits_{x \to \frac{\pi}{2}} \frac{1 - \cos x}{\sin x}$ is equal to
(a) 0 (b) -1
(c) 1 (d) Does not exist

14. Find the mean deviation about the mean for the following data : 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5.
(a) 8.2 (b) 9.2 (c) 6.2 (d) 5.2

15. The equation of the perpendicular bisector of the line segment joining $A(-2, 3)$ and $B(6, -5)$ is
(a) $x - y = -1$ (b) $x - y = 3$
(c) $x + y = 3$ (d) $x + y = 1$

16. If $n(U) = 700$, $n(A) = 200$, $n(B) = 240$, $n(A \cap B) = 100$, then $n(A' \cup B')$ equals
(a) 260 (b) 560
(c) 360 (d) 600

17. Let R be a relation in N defined by $R = \{(x, y) : x + 2y = 8, x, y \in N\}$. The range of R is
(a) $\{2, 4, 6\}$ (b) $\{1, 2, 3\}$
(c) $\{1, 2, 3, 4, 6\}$ (d) $\{4, 5, 6, 7\}$

18. Find the value of $\lim\limits_{x \to 3} \frac{e^x - e^3}{x - 3}$.
(a) e^3 (b) e (c) e^2 (d) 1

19. Solve $5x^2 - 6x + 2 = 0$.
(a) $\frac{3 \pm i}{5}$ (b) $\frac{2 \pm i}{5}$ (c) $\frac{4 \pm i}{5}$ (d) $\frac{1 \pm i}{5}$

20. The mean and variance of n observations $x_1, x_2, x_3, \ldots, x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400$, then the value of n is equal to
(a) 80 (b) 25
(c) 20 (d) 16

SECTION-B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. If a straight line passes through the points $\left(\frac{-1}{2}, 1\right)$ and $(1, 2)$, then its x-intercept is
(a) -2 (b) -1
(c) 2 (d) 1

22. The value of $\left(\frac{1}{3} + 3i\right)^3$ in the form of $a + ib$ is
(a) $1 + 26i$ (b) $\frac{-242}{27} + 26i$
(c) $\frac{-242}{27} - 26i$ (d) $-1 + 26i$

23. In a G.P. of positive terms, if any term is equal to the sum of the next two terms. Then the common ratio of the G.P. is
(a) sin 18° (b) 2cos18°
(c) cos18° (d) 2sin18°

24. $P(A)$ for set $A = \{\{2, 1\}, 0\}$ is
(a) $\{\{\{2, 1\}\}, \{0\}, \{2, 1\}, 0\}$
(b) $\{\{2\}, \{1\}, \{0\}, \phi\}$
(c) $\{\phi, \{\{2, 1\}\}, \{0\}, \{\{2, 1\}, 0\}\}$
(d) None of these

25. If the ordered pairs $(x, -1)$ and $(5, y)$ belong to the set $\{(a, b) : b = 2a - 3\}$, then find the values of x and y.
(a) $x = 1, y = 7$ (b) $x = 7, y = 1$
(c) $x = 3, y = 2$ (d) $x = 2, y = 3$

26. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.
(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$
(c) $\frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$ (d) $\frac{-1 \pm \sqrt{19}i}{\sqrt{5}}$

……

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(a)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(c)
6.	(a)	**7.**	(a,b,c)	**8.**	(c)	**9.**	(a,d)	**10.**	(b,c,d)
11.	(a,d)	**12.**	(a,b)	**13.**	(c)	**14.**	(a)	**15.**	(a)
16.	(c)	**17.**	(0)	**18.**	(4)	**19.**	(2)	**20.**	(126)

27. The following set in roster form is equal to

$A=\left\{x : x=\dfrac{n-2}{n+1}, n\le 7, n\in W\right\}.$

(a) $\left\{-2, \dfrac{-1}{2}, \dfrac{1}{4}, \dfrac{2}{5}, \dfrac{3}{6}, \dfrac{4}{7}, \dfrac{5}{8}\right\}$

(b) $\left\{-2, \dfrac{-1}{2}, 0, \dfrac{1}{4}, \dfrac{2}{5}, \dfrac{3}{6}, \dfrac{4}{7}, \dfrac{5}{8}\right\}$

(c) $\left\{\dfrac{-1}{2}, \dfrac{1}{4}, \dfrac{2}{5}, \dfrac{5}{8}\right\}$ (d) None of these

28. Solve $x^2 - 14x + 58 = 0$.

(a) $\sqrt{7} \pm 3i$ (b) $7 \pm \sqrt{3}\,i$

(c) $7 \pm 3i$ (d) $\sqrt{7} \pm \sqrt{3}\,i$

29. If $f(x) = 4x - 3$ and $g(x) = 5x^2 - 4x + 7$, then find $(f - g)(x)$.

(a) $5x^2 - 8x + 10$ (b) $5x^2 + 8x - 10$

(c) $-5x^2 + 8x - 10$ (d) $-5x^2 - 8x - 10$

30. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

(a) 2042 (b) 2046 (c) 2044 (d) 2045

31. Find $A \cap (B \cup C)$ if $A = \{1, 3, 5, 8\}$, $B = \{3, 5, 7\}$ and $C = \{2, 4, 6, 8\}$.

(a) $\{3, 5, 8\}$ (b) $\{4, 5, 7\}$

(c) $\{2, 3, 6\}$ (d) $\{5, 7, 8\}$

32. The range of the function $f(x)=\dfrac{x-2}{2-x}$ is

(a) R (b) $R - \{1\}$

(c) $\{-1\}$ (d) $R - \{-1\}$

33. The smallest positive integer n, for which $(1 + i)^{2n} = (1 - i)^{2n}$ is

(a) 1 (b) 2

(c) 4 (d) 0

34. Find the value of $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx}; a, b \ne 0.$

(a) $a + b$ (b) $a - b$

(c) ab (d) a/b

35. If $1 - i$, is a root of the equation $x^2 + ax + b = 0$, where $a, b \in R$, then find the values of a and b.

(a) 2, 2 (b) −2, 2 (c) −2, −2 (d) 1, 2

36. Variance of the data, 2, 4, 5, 6, 8, 17 is 23.33. Then variance of 4, 8, 10, 12, 16, 34 will be

(a) 23.23 (b) 25.33

(c) 93.32 (d) 48.66

37. If x, y, z are positive integers, then value of expression $(x + y)(y + z)(z + x)$ is

(a) $\ne 8xyz$ (b) $\ge 8xyz$

(c) $\le 8xyz$ (d) $4xyz$

38. The range of the following set of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is

(a) 11 (b) 7

(c) 5.5 (d) 6

39. $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$ is equal to

(a) 0 (b) 1

(c) 2 (d) − 1

40. Find the mean deviation about the mean of the following data.

Size (x)	1	3	5	7	9	11	13	15
Frequency (f)	3	3	4	14	7	4	3	4

(a) 2.95 (b) 2.35

(c) 3.05 (d) 3.67

SECTION-C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

41. The set of all integers 'x' such that $|x - 3| < 2$ is equal to

(a) $\{1, 2, 3, 4, 5\}$ (b) $\{1, 2, 3, 4\}$

(c) $\{2, 3, 4\}$ (d) $\{-4, -3, -2\}$

42. What is the domain of the real valued function $f(x)=\dfrac{1}{3x-2}$?

(a) R (b) $R=\left\{\dfrac{4}{3}\right\}$

(c) $R-\left\{\dfrac{2}{3}\right\}$ (d) $\left\{\dfrac{2}{3}\right\}$

43. The standard deviation of a distribution is 30 and each item is raised by 3, then new S.D. is

(a) 32 (b) 28 (c) 27 (d) 30

44. $\lim\limits_{x\to -1}\dfrac{x^{10}+x^5+1}{x-1}$ is equal to

(a) $\dfrac{1}{2}$ (b) −1 (c) $-\dfrac{1}{2}$ (d) −2

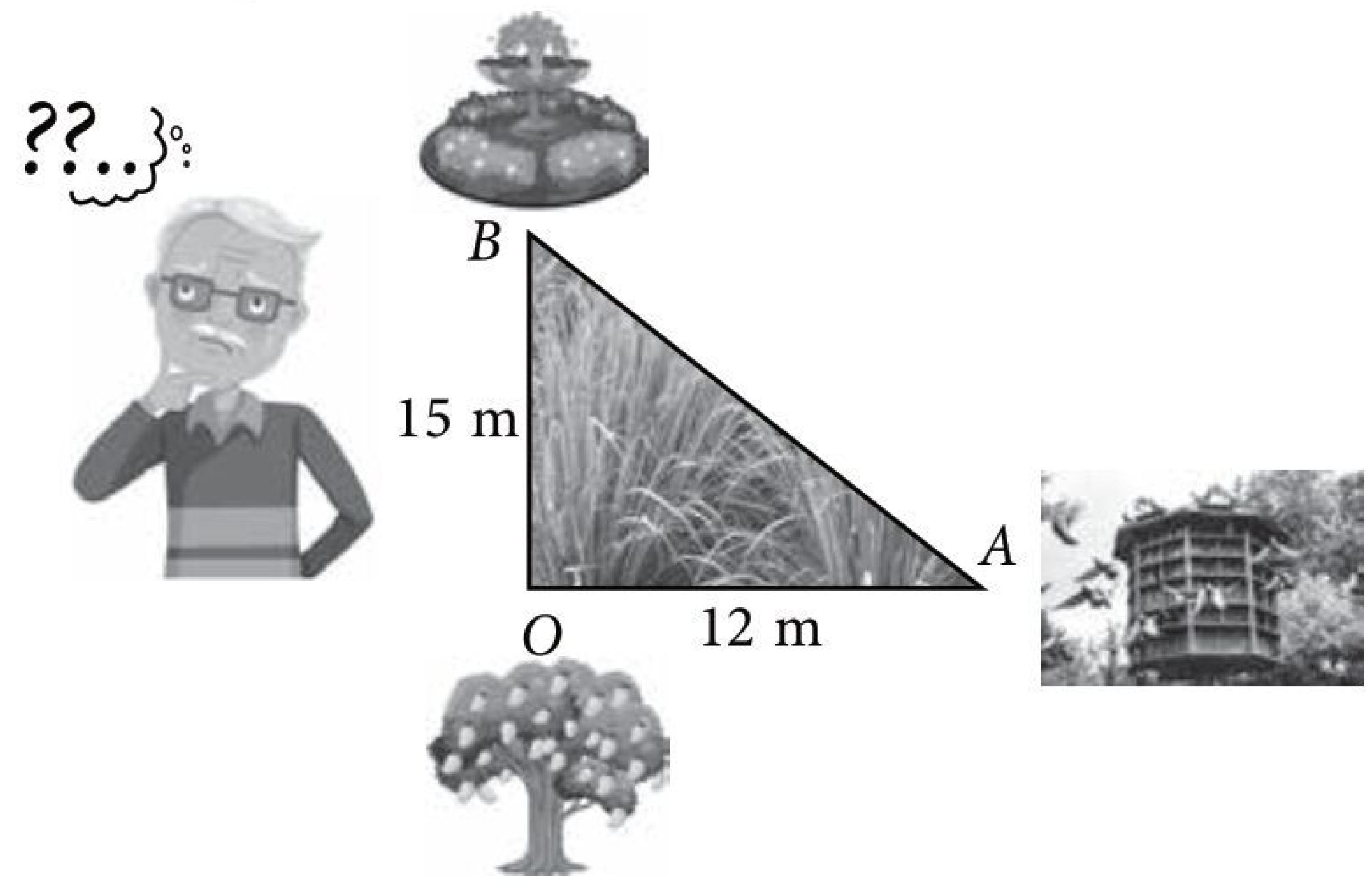
45. A set of n values $x_1, x_2, \ldots, x_n$ has mean $\bar{x}$. The mean of n values $x_1 + \bar{x}, x_2 + \bar{x}, \ldots, x_n + \bar{x}$ will be

(a) $\bar{x}$ (b) $2\bar{x}$

(c) $(n-1)\bar{x}$ (d) 0

CASE STUDY

Ramakrishna has a triagular space in the backyard of his house. He wants to plant a mango tree at one corner. At a distance of 12 m towards east from mango tree, he wants to make a pigeon house and at a distance of 15 m towards north from mango tree, he feels the need of water fountain to make his backyard beautiful as shown in figure.



Based on the given information, answer the following questions.

46. If mango tree is considered at origin, then find the coordinates of point where he wants to make pigeon house and water fountain respectively.

(a) (0, 0), (12, 15)	(b) (0, 12), (15, 0)
(c) (12, 0), (0, 15)	(d) (12, 15), (15, 12)

47. Find the equation of line AB.

(a) $4x + 5y = 20$	(b) $5x + 4y = 60$
(c) $5x + 4y = 50$	(d) $4x + 5y = 60$

48. Find the perpendicular distance of line AB from mango tree.

(a) $\frac{60}{\sqrt{41}}$ m	(b) $\frac{59}{\sqrt{41}}$ m
(c) 20 m	(d) 61 m

49. Slope of line AB is

(a) $\frac{4}{5}$	(b) $\frac{-5}{4}$
(c) $\frac{-4}{5}$	(d) $\frac{5}{4}$

50. Length of line AB is

(a) 20 m	(b) 17.32 m
(c) 28.66 m	(d) 19.21 m

SOLUTIONS

1. (c) : Multiplicative inverse of

$$\frac{3+4i}{4-5i} = \frac{4-5i}{3+4i} = \frac{4-5i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{(12-20)+i(-15-16)}{9+16} = -\frac{8}{25} - i\frac{31}{25}$$

2. (d) : Slope of the line passing through the points $(-2, 6)$ and $(4, 8)$ is $m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

Slope of the line passing through the points $(8, 12)$ and $(x, 24)$ is $m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since, two lines are perpendicular.

$\therefore \quad m_1 m_2 = -1 \Rightarrow \frac{1}{3} \times \frac{12}{x-8} = -1$

$\Rightarrow \quad x = 4$

3. (a) : The given equality can be rewritten as

$(x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)$

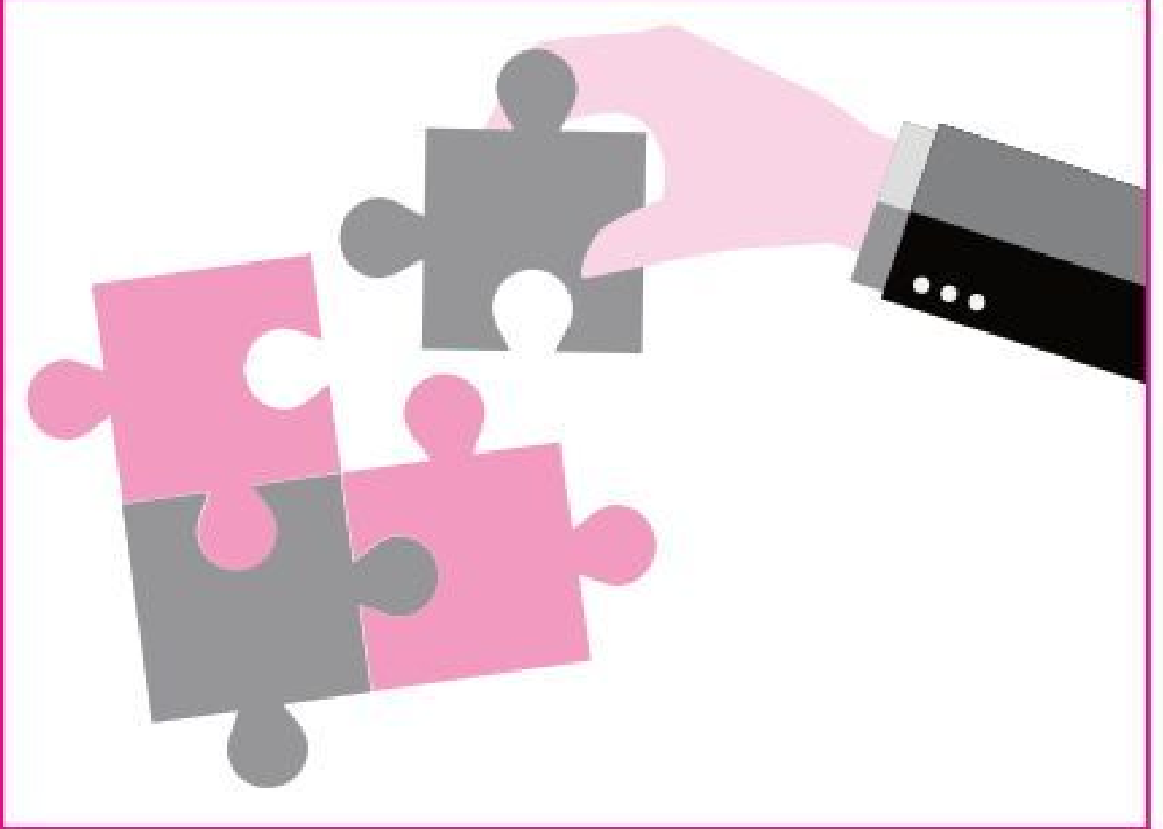
$\Rightarrow \quad x^4 - 3x^2 = 4,\ 2x - y = 2y - 5$

$\Rightarrow \quad (x^2 - 4)(x^2 + 1) = 0,\ y = \frac{2x+5}{3}$

$\Rightarrow \quad x = \pm 2 \quad (\because\ x^2 \neq -1)$

$\therefore \quad$ At $x = 2$, $y = 3$ and at $x = -2$, $y = 1/3$

PUZZLE CORNER

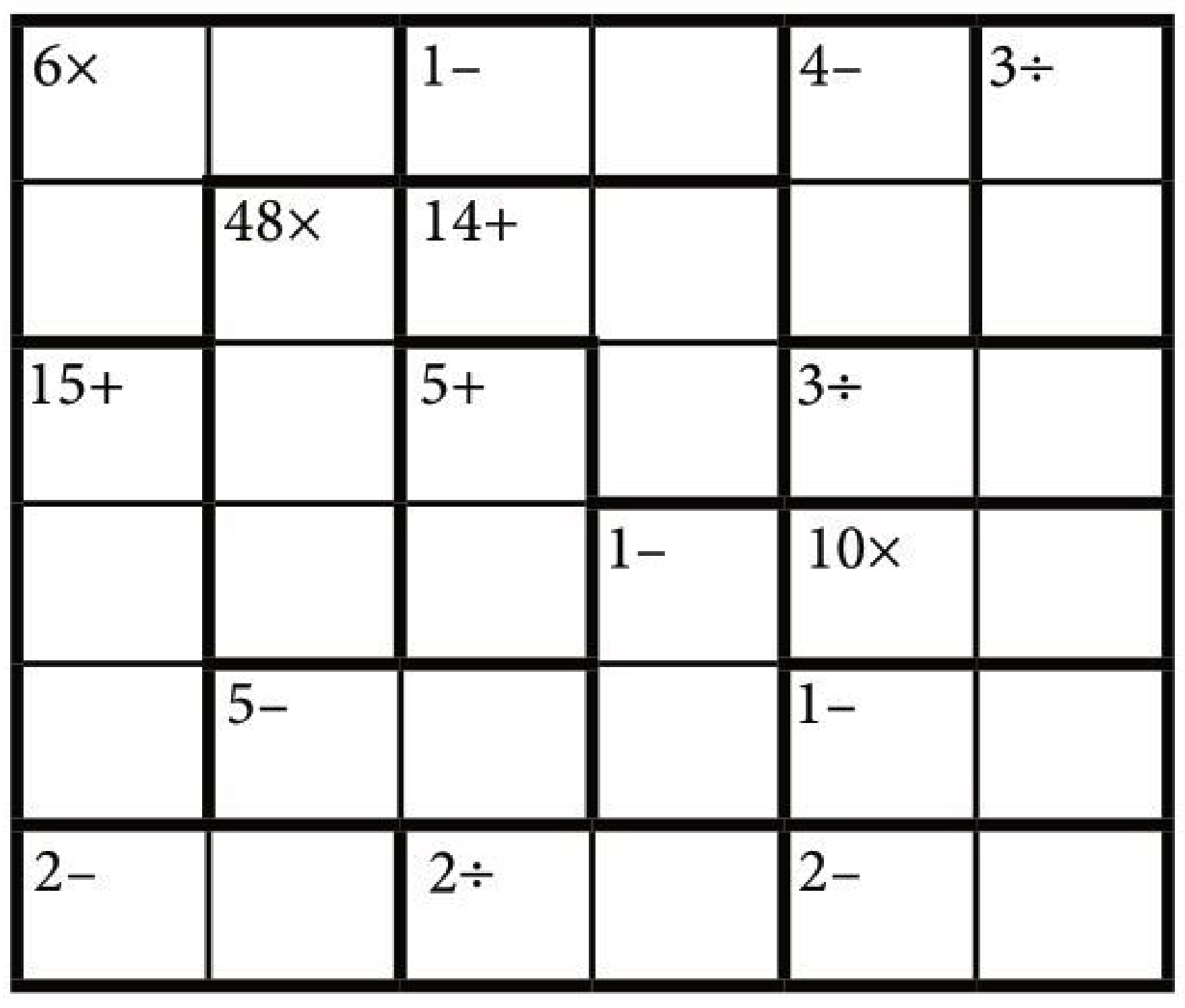


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



6×		1–		4–	3÷
	48×	14+			
15+		5+		3÷	
			1–	10×	
	5–			1–	
2–		2÷		2–	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

4. (d) : Let $a_1, a_2, \ldots, a_n$ be the given A.P., then

$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n = nP + \frac{1}{2}n(n-1)Q$

Therefore, $S_1 = a_1 = P$, $S_2 = a_1 + a_2 = 2P + Q$
so that $a_2 = S_2 - S_1 = P + Q$
Hence, the common difference is given by $d = a_2 - a_1$
$= (P + Q) - P = Q$.

5. (c) : Element of A is x which must have properties : It is an odd integer and is among 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. So they are 3, 5, 7, 9, 11, 13

$\therefore \quad A = \{3, 5, 7, 9, 11, 13\}$

6. (c) : $n(A) = p$, $n(B) = q$, $n(A \times B) = pq = 7$
So possible values of p, q are 7, 1
$\Rightarrow \quad p^2 + q^2 = (7)^2 + (1)^2 = 50$

7. (a) : Consider, $\lim\limits_{x\to 1} \dfrac{x^{1/3}-1}{x^{1/6}-1} = \lim\limits_{x\to 1} \dfrac{\left(x^{1/6}\right)^2-1}{x^{1/6}-1}$

$= \lim\limits_{x\to 1}\left(x^{1/6}+1\right) = 1+1 = 2$

8. (c) : $i^9 + i^{19} = i \cdot i^8 + i \cdot i^{18} = i(i^2)^4 + i(i^2)^9$
$= i(-1)^4 + i(-1)^9$
$= i + i(-1) = i - i = 0 + i0 = a + ib$, where $a = 0$, $b = 0$.

9. (b) : Number of subsets of a set having 'n' elements is 2^n.

10. (c) : Here, $b^2 - 4ac = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3$
Therefore, the solutions are given by

$$x = \frac{-1 \pm \sqrt{-3}}{2 \times 1} = \frac{-1 \pm \sqrt{3}i}{2}$$

11. (a) : Given, $A = \{x : |x| < 3, x \in I\}$
$\Rightarrow \quad A = \{x : -3 < x < 3, x \in I\} = \{-2, -1, 0, 1, 2\}$
Also, $R = \{(x, y) : y = |x|\}$
$\therefore \quad R = \{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$

12. (a) : Here, first term $a = 3$ and common difference $d = 4$. Let there be n terms in the given A.P. Then, n^{th} term $= 407 \Rightarrow 3 + (n - 1) \times 4 = 407 \Rightarrow n = 102$
So, there are 102 terms in the given A.P.

13. (c) : We have, $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{1-\cos x}{\sin x}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} = \lim_{x\to\frac{\pi}{2}} \tan\left(\frac{x}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

14. (c) : The mean of the given data is,

$$\bar{x} = \frac{1}{20}\sum_{i=1}^{20} x_i = \frac{200}{20} = 10$$

The values of the deviations from mean, *i.e.*, $|x_i - \bar{x}|$ are 2, 7, 8, 7, 6, 1, 7, 9, 10, 5, 2, 7, 8, 7, 6, 1, 7, 9, 10, 5

$$\therefore \quad \sum_{i=1}^{20}|x_i - \bar{x}| = 124 \text{ and M.D. } (\bar{x}) = \frac{124}{20} = 6.2$$

15. (b) : Mid point of given line segment AB is $(2, -1)$.
Now, slope of the line segment AB is $-8/8 = -1$
Slope of the required line segment is 1.
So equation of the required line is
$y + 1 = 1(x - 2) \Rightarrow x - y = 3$.

16. (d) : $n(A' \cup B') = n((A \cap B)')$
$= n(U) - n(A \cap B) = 700 - 100 = 600$

17. (b) : $R = \{(x, y) : x + 2y = 8, x, y \in N\}$

$x + 2y = 8 \Rightarrow y = \dfrac{8-x}{2}$

$x = 1 \Rightarrow y = \dfrac{7}{2} \notin N;\ x = 2 \Rightarrow y = 3 \in N$

$x = 3 \Rightarrow y = \dfrac{5}{2} \notin N;\ x = 4 \Rightarrow y = 2 \in N$

$x = 5 \Rightarrow y = \dfrac{3}{2} \notin N;\ x = 6 \Rightarrow y = 1 \in N$

$x = 7 \Rightarrow y = \dfrac{1}{2} \notin N;\ x = 8 \Rightarrow y = 0 \notin N$

$\therefore \quad R = \{(2, 3), (4, 2), (6, 1)\}$
$\therefore \quad$ Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$

18. (a) : $\lim\limits_{x\to 3} \dfrac{e^x - e^3}{x-3} = \lim\limits_{x\to 3} e^3\left(\dfrac{e^{x-3}-1}{x-3}\right)$

$= \lim\limits_{x-3\to 0} e^3\left(\dfrac{e^{x-3}-1}{x-3}\right) \quad [\because x \to 3 \Rightarrow x - 3 \to 0]$

$= e^3 \times 1 = e^3$

19. (a) : Given, $5x^2 - 6x + 2 = 0$
$D = (-6)^2 - 4(5)(2) = 36 - 40 = -4$

$$\therefore \quad x = \frac{-(-6) \pm \sqrt{-4}}{2(5)} = \frac{6 \pm 2i}{10} = \frac{3 \pm i}{5}$$

20. (d) : Variance $= \sigma^2$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = \sigma^2$$

$$\Rightarrow \quad 0 = \frac{1}{n}(400) - (5)^2 \Rightarrow \frac{400}{n} = 25 \Rightarrow n = 16$$

21. (a) : Slope $= \dfrac{(2-1)}{\left\{1-\left(-\frac{1}{2}\right)\right\}} = \dfrac{1}{\left(\frac{3}{2}\right)} = \dfrac{2}{3}$

So, equation of the line, $y - 2 = \dfrac{2}{3}(x-1)$

$\Rightarrow \quad y=\frac{2}{3}x+\frac{4}{3} \;\Rightarrow\; \frac{x}{-2}+\frac{y}{4/3}=1$

$\therefore$ x-intercept = – 2.

22. (c) : $\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+3\left(\frac{1}{3}\right)^2(3i)+3\left(\frac{1}{3}\right)(3i)^2+(3i)^3$

$=\frac{1}{27}+i+9(-1)+27(-i)=\frac{1}{27}-9-26i$

$=\frac{1-243}{27}-26i=\frac{-242}{27}-26i$

23. (d) : We have, $t_n = t_{n+1} + t_{n+2}$

$\Rightarrow \quad ar^{n-1} = ar^n + ar^{n+1} \;\Rightarrow\; 1 = r + r^2$

$\Rightarrow \quad r=\frac{-1\pm\sqrt{5}}{2}$, since $r > 0$

Therefore, $r=2\left(\frac{\sqrt{5}-1}{4}\right)=2\sin 18^\circ$

24. (c) : $P(A) = \{\phi, \{\{2, 1\}\}, \{0\}, \{\{2, 1\}, 0\}\}$

25. (a) : Given, $(x, -1) \in \{(a, b): b = 2a - 3\}$

$\therefore \quad -1 = 2x - 3 \;\Rightarrow\; 2x = 2 \;\Rightarrow\; x = 1$

Also, $(5, y) \in \{(a, b): b = 2a - 3\}$

$\therefore \quad y = (2 \times 5) - 3 \;\Rightarrow\; y = 10 - 3 \Rightarrow y = 7$

So, $x = 1, y = 7$

26. (c) : Here, $b^2-4ac=1^2-4\times\sqrt{5}\times\sqrt{5}=1-20=-19$

$\therefore \quad x=\frac{-1\pm\sqrt{-19}}{2\sqrt{5}}=\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

27. (b) : $A=\left\{x: x=\frac{n-2}{n+1}, n\le 7, n\in W\right\}$

Given, $n \le 7$ and $n \in W$. So, $n = 0, 1, 2, 3, 4, 5, 6, 7$

$\therefore \quad A=\left\{-2, \frac{-1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \frac{5}{8}\right\}$

28. (c) : $x=\frac{-(-14)\pm\sqrt{(-14)^2-4(1)(58)}}{2}$

$=\frac{14\pm\sqrt{196-232}}{2}=\frac{14\pm 6i}{2}=7\pm 3i$

29. (c) : Here, $f(x) = 4x - 3$, $g(x) = 5x^2 - 4x + 7$

$(f-g)(x) = f(x) - g(x) = (4x - 3) - (5x^2 - 4x + 7)$

$= -5x^2 + 8x - 10$

30. (b) : Here $a = 2$, $r = 2$ and $n = 10$

$\therefore \quad S_n=\frac{a(r^n-1)}{r-1} \;\Rightarrow\; S_{10}=2(2^{10}-1)=2046$

Hence, the number of ancestors preceding the person is 2046.

31. (a) : $A\cap(B\cup C)=\{1,3,5,8\}\cap[\{3,5,7\}\cup\{2,4,6,8\}]$

$=\{1, 3, 5, 8\}\cap\{2, 3, 4, 5, 6, 7, 8\}$

$=\{3, 5, 8\}$

32. (c) : $f(x)=\frac{x-2}{2-x}=-\frac{x-2}{x-2}=-1$, if $x\ne 2$

$\therefore$ Range of $f = \{-1\}$

33. (b) : We have, $(1 + i)^{2n} = (1 - i)^{2n} \Rightarrow \left(\frac{1+i}{1-i}\right)^{2n}=1$

$\Rightarrow \quad (i)^{2n} = 1.$

Thus, the smallest positive integer $n = 2$. $(\because i^4 = 1)$

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34. (d) : Given, $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx}; a, b \neq 0$

$$= \lim_{x\to 0}\frac{\left(\dfrac{\sin ax}{ax}\right)\times ax}{bx} = \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)\times\frac{ax}{bx} = 1\times\frac{a}{b} = \frac{a}{b}$$

35. (b) : Since complex roots always occur in conjugate pair.

∴ Other root is $1 + i$.

Sum of roots $= \dfrac{-a}{1} = (1 - i) + (1 + i) \Rightarrow a = -2,$

Products of roots $= \dfrac{b}{1} = (1 - i)(1 + i) \Rightarrow b = 2$

36. (c) : When each observation is multiplied by 2, then variance is also multiplied by 2^2.

∴ New Variance $= 2^2 \times 23.33 = 93.32$

37. (b) : We know that, A.M. ≥ G.M.

$\Rightarrow \quad \dfrac{x+y}{2} \geq \sqrt{xy}, \dfrac{y+z}{2} \geq \sqrt{yz}$ and $\dfrac{z+x}{2} \geq \sqrt{zx}$

Multiplying the three inequalities, we get

$$\frac{x+y}{2}\cdot\frac{y+z}{2}\cdot\frac{x+z}{2} \geq \sqrt{(xy)(yz)(zx)}$$

$\Rightarrow \quad (x + y)(y + z)(z + x) \geq 8xyz$

38. (b) : Range $= 9 - 2 = 7$

39. (b) : We have, $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}\right)$

$$= \lim_{x\to 0}\left\{\frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)}\right\} = \lim_{x\to 0}\left\{\frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x}\right\}$$

$$= \frac{1}{2}\lim_{x\to 0}\{\sqrt{1+x} + \sqrt{1-x}\} = \frac{1}{2}\times 2 = 1$$

40. (a) : Mean, $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$= \frac{3+9+20+98+63+44+39+60}{42} = \frac{336}{42} = 8$$

$$\text{M.D.}(\bar{x}) = \frac{\Sigma f_i\,|x_i - \bar{x}|}{\Sigma f_i}$$

$$= \frac{3(7)+3(5)+4(3)+14(1)+7(1)+4(3)+3(5)+4(7)}{42}$$

$$= \frac{21+15+12+14+7+12+15+28}{42} = \frac{62}{21} = 2.95$$

41. (c) : $|x - 3| < 2$

$\Rightarrow \quad 3 - 2 < x < 3 + 2$

$\Rightarrow \quad 1 < x < 5$

$\Rightarrow \quad x = 2, 3, 4$

∴ Required set $= \{2, 3, 4\}$

42. (c) : Given, $f(x) = \dfrac{1}{3x-2}$

$f(x)$ assumes real values, for all real values of x except for $3x - 2 = 0$ *i.e.*, $x = \dfrac{2}{3}$

∴ Domain of $f(x) = R - \left\{\dfrac{2}{3}\right\}$

43. (d) : S.D. of a series unaltered if each item is raised (reduced) by the some scalar quantity. So S.D. is independent of change of origin.

∴ New S.D. = 30

44. (c) : $\lim\limits_{x\to -1}\dfrac{x^{10} + x^5 + 1}{x - 1} = \dfrac{1 - 1 + 1}{-1 - 1} = -\dfrac{1}{2}$

45. (b) : $(x_1 + \bar{x}) + (x_2 + \bar{x}) + \ldots + (x_n + \bar{x})$

$$= (x_1 + x_2 + x_3 + \ldots + x_n) + \underbrace{(\bar{x} + \bar{x} + \ldots. + \bar{x})}_{n \text{ times}}$$

$$= n\bar{x} + n\bar{x} = 2n\bar{x}$$

∴ Required mean $= \dfrac{2n\bar{x}}{n} = 2\bar{x}$

46. (c) : Pigeon house is at point A, which is at a distance of 12 m from mango tree and water fountain is at point B, which is at a distance of 15 m from mango tree.

∴ Required coordinates are $A(12, 0)$ and $B(0, 15)$.

47. (b) : By intercept form, equation of line AB is

$$\frac{x}{12} + \frac{y}{15} = 1$$

$\Rightarrow \quad 5x + 4y = 60$

48. (a) : Perpendicular distance of AB from mango tree (origin) is

$$\left|\frac{0 + 0 - 60}{\sqrt{25+16}}\right| = \frac{60}{\sqrt{41}} \text{ m}$$

49. (b) : Equation of line AB is $5x + 4y = 60$

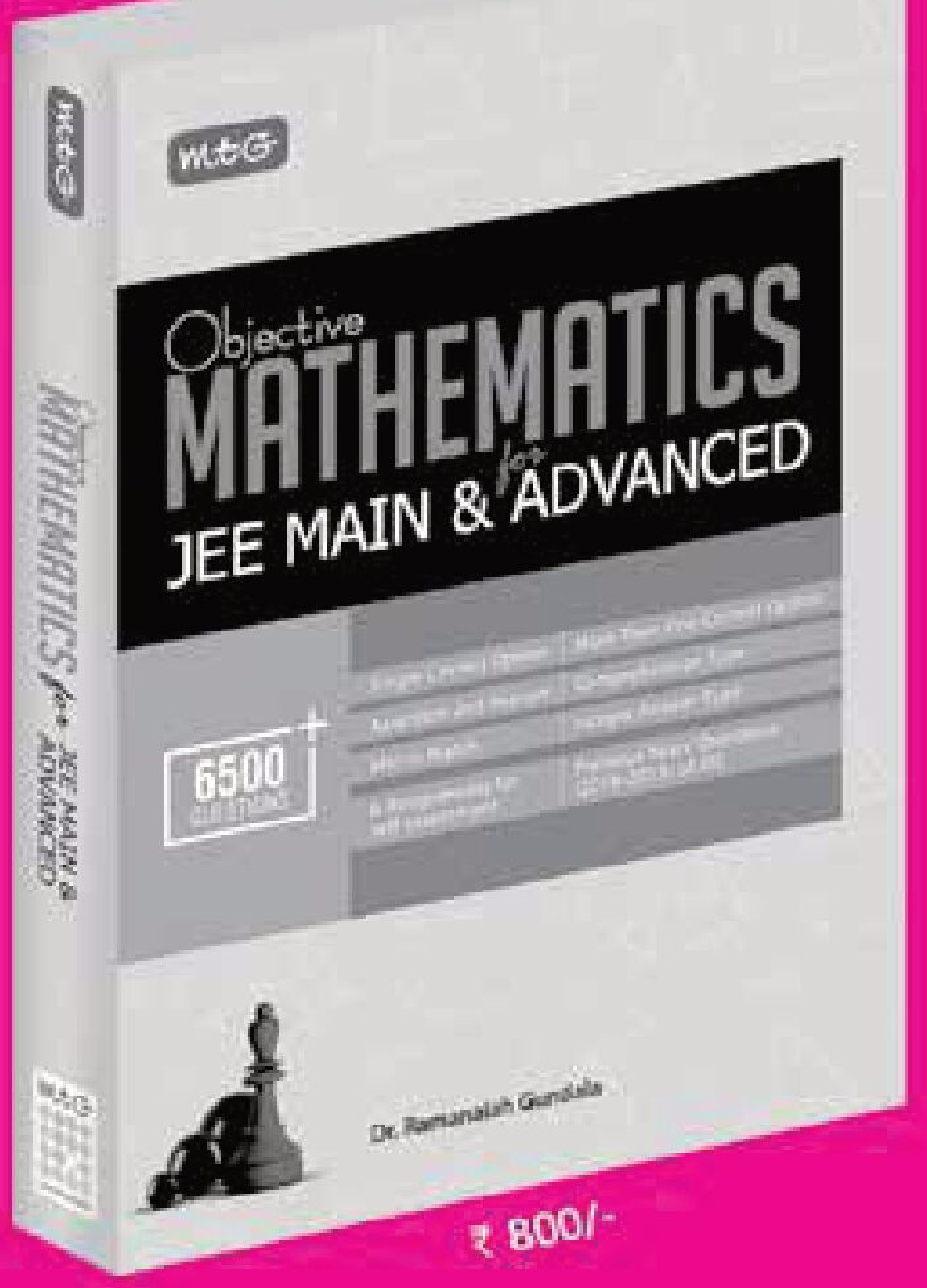
$\Rightarrow \quad y = \dfrac{-5}{4}x + 15$

∴ Slope of line is $\dfrac{-5}{4}$.

50. (d) : Length of line AB

$$= \sqrt{144 + 225} = \sqrt{369} = 19.21 \text{ m}$$

ABRACADABRA



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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Binomial Theorem

Time Taken : 60 Min.

Only One Option Correct Type

1. Given the positive integers $r > 1$, $n > 2$ and the coefficients of $(3r)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1+x)^{2n}$ are equal, then $n =$

(a) $2r$ (b) $2r-1$ (c) $2r+1$ (d) $3r$

2. Middle term in the expansion of $(1-3x+3x^2-x^3)^{2n}$ is

(a) $\frac{6n!}{3n!\,3n!}x^n$ (b) $\frac{6n!}{3n!}x^{3n}$

(c) $\frac{6n!}{3n!\,3n!}(-x)^{3n}$ (d) None of these

3. If $a^2 + b^3 = 4$, then the maximum value of the term independent of x in the expansion of $(ax^{1/2} + bx^{-1/3})^5$ $\forall\ a, b > 0$, is

(a) 80 (b) 60 (c) 40 (d) 10

4. If $T_0, T_1, T_2, \ldots, T_n$ represents the terms in the expansion of $(x+a)^n$, then the value of

$(T_0 - T_2 + T_4 - T_6 + \ldots)^2 + (T_1 - T_3 + T_5 - T_7 + \ldots)^2$

is equal to

(a) $(x^2 - a^2)^n$ (b) $(x^2 + a^2)^n$

(c) $(x+a)^n - (x-a)^n$ (d) $(x+a)^n + (x-a)^n$

5. If the coefficients of $x^2y^5z^3$ in the expansion of $(2x + y + 3z)^{10}$ is $k \cdot 10! \times 3^3$, then k equals

(a) $\frac{10!}{2!\,5!\,3!}\ 2^2\ 3^2$ (b) $\frac{10!}{5!\,3!}\ 3^3\ 2^2$

(c) $\frac{1}{3 \cdot 5!}$ (d) $\frac{10!}{2!\,5!\,3!}\ 2^2\ 3^3$

6. If 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x+A)^n$ are respectively 84, 280 and 560, then the set of values of x, A and n are respectively

(a) (1, 2, 7) (b) (1, 2, 5)

(c) (2, 1, 5) (d) (7, 2, 5)

One or More Than One Option(s) Correct Type

7. If m be the number of terms in the expansion of $(1+x)^{51}(1-x+x^2)^{50}$ and if unit's place & ten's place digit in 9^m are 'a' and 'b', then

(a) $a + b = 9$ (b) $a^2 + b^2 = 65$

(c) $(a-b)^2 = 49$ (d) None of these

8. If $x + \frac{1}{x} = -1$ and $\alpha = x^{2018} + \frac{1}{x^{2018}}$ and β be the digit at unit place in the number $5^{5^n} + 1$, $n \in N$ and $n > 1$, then $\alpha + 2\beta$ equals

(a) 5 (b) 6

(c) 11 (d) 12

9. If the third term in the expansion of $\left(x + x^{\log_{10} x}\right)^5$ is 10^6, then $x =$

(a) 10 (b) 100

(c) 10^{-3} (d) $10^{-5/2}$

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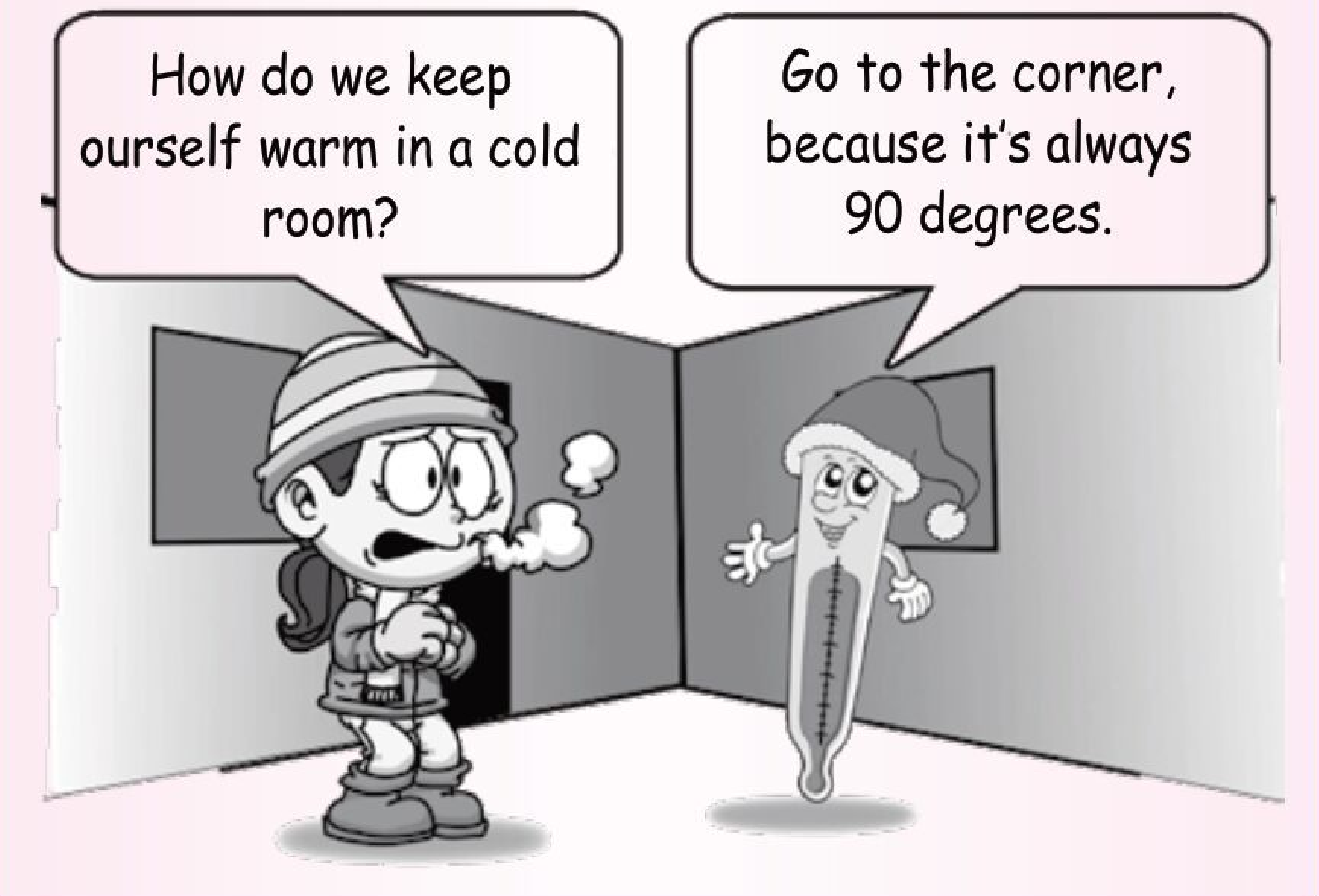
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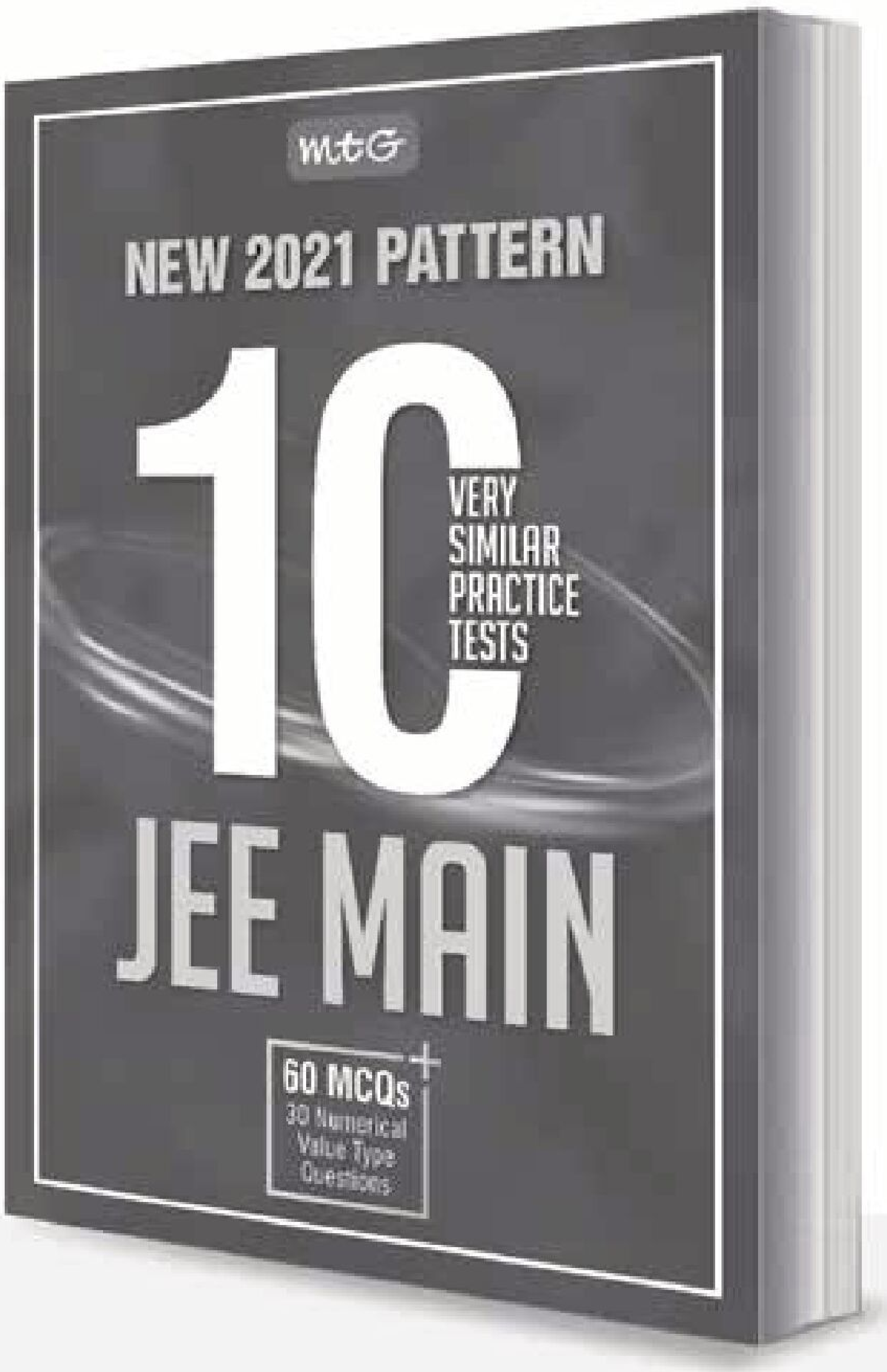
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10. If expansion of $\left(\frac{5}{x^2}+x^4\right)^n, n \in N$ has a term independent of x, then the value of n can be

(a) 20 (b) 24
(c) 63 (d) 75

11. The remainder when $\frac{4^k+5}{3}(k \in N)$ is divided by 10, is

(a) 7 for $k = 2m$ (b) 9 for $k = 2m$
(c) 2 for $k = 2m - 1$ (d) 3 for $k = 2m - 1$

12. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ and $\sum_{r=0}^{n}(r^2-3r+1)C_r = 2^{n-2}f(n)$, if roots of $f(x) = 0$ are α and β, then

(a) $2\alpha + 3\beta = 14$ or 11
(b) $\alpha^3 + \beta^3 = 65$
(c) $\alpha^3 + \beta^3 = 64$
(d) $2\alpha + 3\beta = 10$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let $\sum_{r=0}^{20} a_r x^r = (1+x+x^2)^{10}$. Then

13. $a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13} + a_{15} + a_{17} + a_{19} =$

(a) $\frac{1}{2}$ (b) 3^{10}
(c) $\frac{1}{2}\left(3^{10}-1\right)$ (d) $\frac{1}{2}(3^{10}+1)$

14. $a_0 + a_3 + a_6 + a_9 + a_{12} + a_{15} + a_{18} =$

(a) 3^9 (b) $3^9 - 1$
(c) $3^9 + 1$ (d) $3^{10} - 1$

15. $a_5 - \binom{10}{1}a_4 + \binom{10}{2}a_3 - \binom{10}{3}a_2 + \binom{10}{4}a_1 - \binom{10}{5}a_0 =$

(a) 0 (b) $\binom{10}{4}$
(c) $-\binom{10}{5}$ (d) $\binom{10}{2}$

Matrix Match Type

16. Match the following :

	Column-I	Column-II	
P.	The remainder when 17^{20} is divided by 81, is	1.	60
Q.	If $f(n) = \sum_{s=1}^{n}\sum_{r=s}^{n} {}^nC_r\, {}^rC_s$, then $f(3) =$	2.	19
R.	The coefficient of x^2y in the expansion of $(1 + x + 2y)^5$ is equal to	3.	55
S.	If $C_r = \binom{10}{r}$, then $\sum_{r=1}^{10} \frac{r \cdot C_r}{C_{r-1}}$ is equal to	4.	46

	P	Q	R	S
(a)	1	2	3	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	2	3	1

Numerical Answer Type

17. If $2^{2007} + 2008$ is divided by 9, then the remainder is______.

18. The last digit of the coefficient of x^2 in the product $(1-x+2x^2)\left(x+\frac{1}{x}\right)^{10}$ is______.

19. If $\frac{1}{\sqrt{4x+1}}\left[\left(\frac{1+\sqrt{4x+1}}{2}\right)^n - \left(\frac{1-\sqrt{4x+1}}{2}\right)^n\right]$ is a fifth degree polynomial, then the sum of the digits of the integer n is______.

20. If the sum of the coefficients in the expansion of $(x - 3y + 5z)^n$ is 19683, then the greatest coefficient in the expansion of $(1 + x)^n$ is______.

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Exam on
6th December
2021

TERM-I OBJECTIVE TYPE QUESTIONS

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

SECTION - A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1. The value of $\sin^{-1}\left\{\sin\frac{5\pi}{6}\right\}+\tan^{-1}\left\{\tan\frac{\pi}{6}\right\}=$

(a) $\pi/6$ (b) $2\pi/3$
(c) $\pi/3$ (d) $5\pi/6$

2. If $f(x)=\begin{cases}6x^5, & x\le 0\\ 2a+x, & x>0\end{cases}$ is continuous at $x = 0$, then $a =$

(a) 1 (b) 0
(c) -1 (d) 2

3. If $[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 2 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\\ x\end{bmatrix}=O$, then the values of x are

(a) 1, 5 (b) $-1, -5$
(c) 1, 6 (d) $-1, -6$

4. Compute the product $\begin{bmatrix}-1 & 3 & 0\\ 2 & 1 & 4\end{bmatrix}\times\begin{bmatrix}1 & 3\\ 0 & 2\\ -2 & -1\end{bmatrix}$.

(a) $\begin{bmatrix}1 & 3\\ 6 & -4\end{bmatrix}$ (b) $\begin{bmatrix}-1 & -3\\ 6 & 4\end{bmatrix}$
(c) $\begin{bmatrix}-1 & 3\\ -6 & 4\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 3\\ -6 & -4\end{bmatrix}$

5. The value of x for which $f(x) = x^3 - 6x^2 - 36x + 7$ is strictly increasing, belongs to

(a) $(-1, 0) \cup (1, 5)$ (b) $(-2, 0) \cup (1, 6)$
(c) $(-\infty, -2) \cup (6, \infty)$ (d) $(-2, 6)$

6. For a 2 × 2 matrix A, if $A\cdot\text{adj}\,A=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$, then det A equals

(a) 0 (b) 10 (c) 20 (d) 100

7. Which of the following functions from Z into Z are bijections?

(a) $f(x) = x^3$ (b) $f(x) = x + 2$
(c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + 1$

8. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = A + B$, where A is symmetric and B is skew-symmetric, then $B =$

(a) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9. If slope of tangent to curve $y = x^3$ at a point is equal to ordinate of point, then point is

(a) (27, 3) (b) (3, 27) (c) (1, 2) (d) (–1, 3)

10. The domain of the function $\cos^{-1}(2x - 1)$ is

(a) [0, 1] (b) [–1, 1]

(c) (–1, 1) (d) [0, π]

11. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is

(a) reflexive (b) transitive

(c) symmetric (d) None of these

12. If $y = \frac{x}{x+1} + \frac{x+1}{x}$, then $\frac{d^2y}{dx^2}$ at $x = 1$ is equal to

(a) 7/4 (b) 7/8 (c) 1/4 (d) –7/8

13. If $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$ and $C = (c_{ij})_{p \times q}$, then the product $(BC)A$ is possible only when

(a) $m = q$ (b) $n = q$

(c) $p = q$ (d) $m = p$

14. If $\tan^{-1}(x^2 + y^2) = \alpha$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-x}{y}$ (b) xy (c) $\frac{x}{y}$ (d) $-xy$

15. If $A = \begin{bmatrix} x & x-1 \\ 2x & 1 \end{bmatrix}$ and if det $A = -9$, then the values of x are

(a) $\frac{3}{2}, -3$ (b) $\frac{-2}{3}, 3$ (c) $\frac{2}{3}, 3$ (d) $\frac{-3}{2}, 3$

16. A normal to the curve $2x^2 - y^2 = 14$ at the point (x_1, y_1) is parallel to the straight line $x + 3y = 4$. Then the point (x_1, y_1) is

(a) (3, 3) (b) (– 4, –2)

(c) (2, 3) (d) (3, 2)

17. If $A = \begin{bmatrix} 0 & 3 \\ 4 & 5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 4a \\ 3b & 60 \end{bmatrix}$, then value of k, a and b respectively are

(a) 12, 19, 16 (b) 9, 12, 16

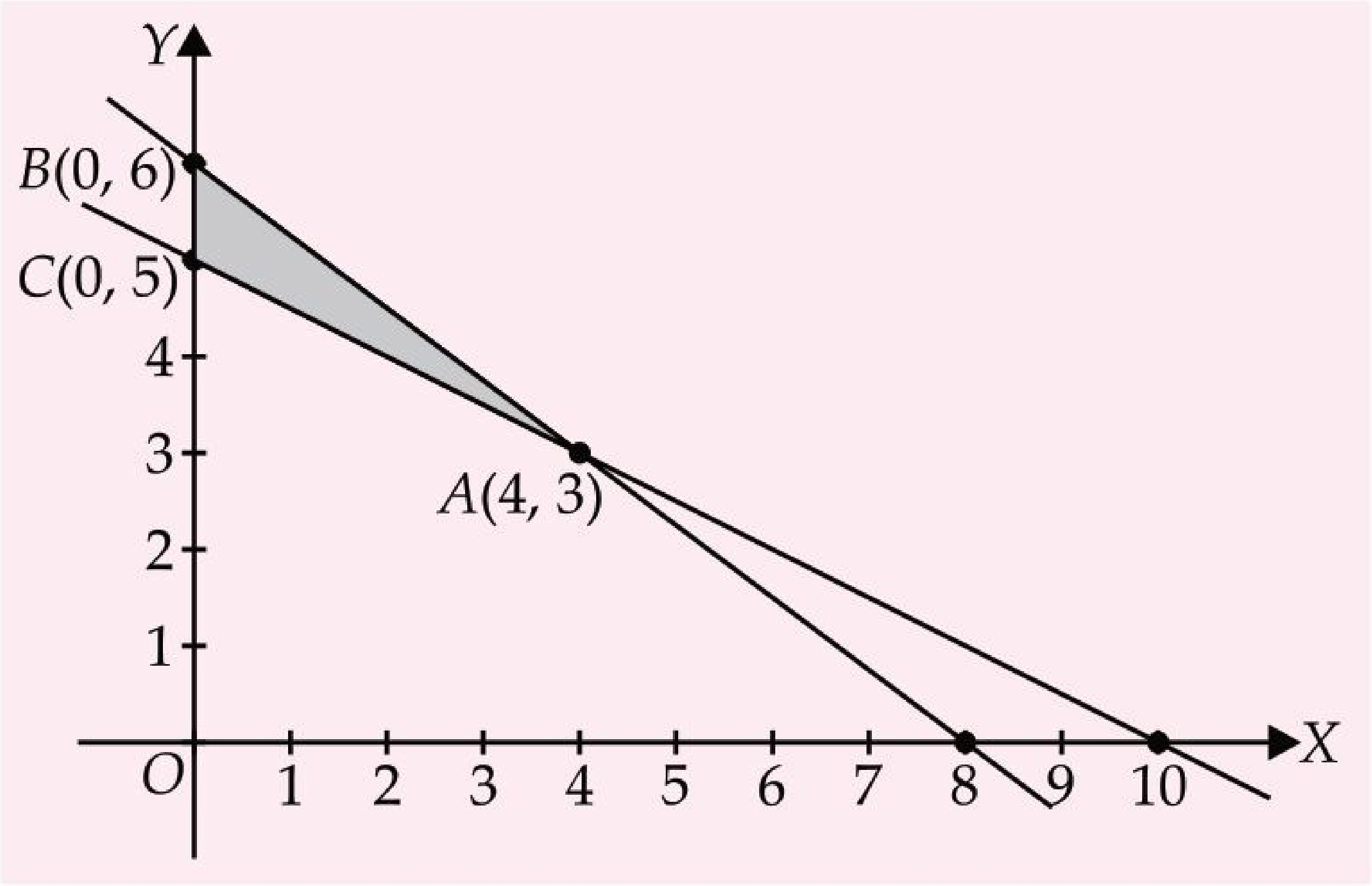
(c) 12, 9, 16 (d) 16, 9, 12

18. If $y = \log(\log x)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{(1+\log x)}{x^2 \log x}$ (b) $\frac{-(1+\log x)}{(x \log x)^2}$

(c) $\frac{(1+\log x)}{(x \log x)^2}$ (d) $\frac{-(1+\log x)}{x^2 \log x}$

19. In the given graph, the feasible region for a L.P.P. is shaded. The minimum value of objective function $z = 2x + 5y$ is



(a) 30 (b) 25 (c) 23 (d) 19

20. The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$, where $k > \frac{3}{2}$ and $x \in R$, is

(a) decreasing (b) strictly increasing

(c) strictly decreasing (c) none of these

SECTION - B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. The domain of the function $f(x) = \sin^{-1}\left(\frac{x+5}{2}\right)$ is

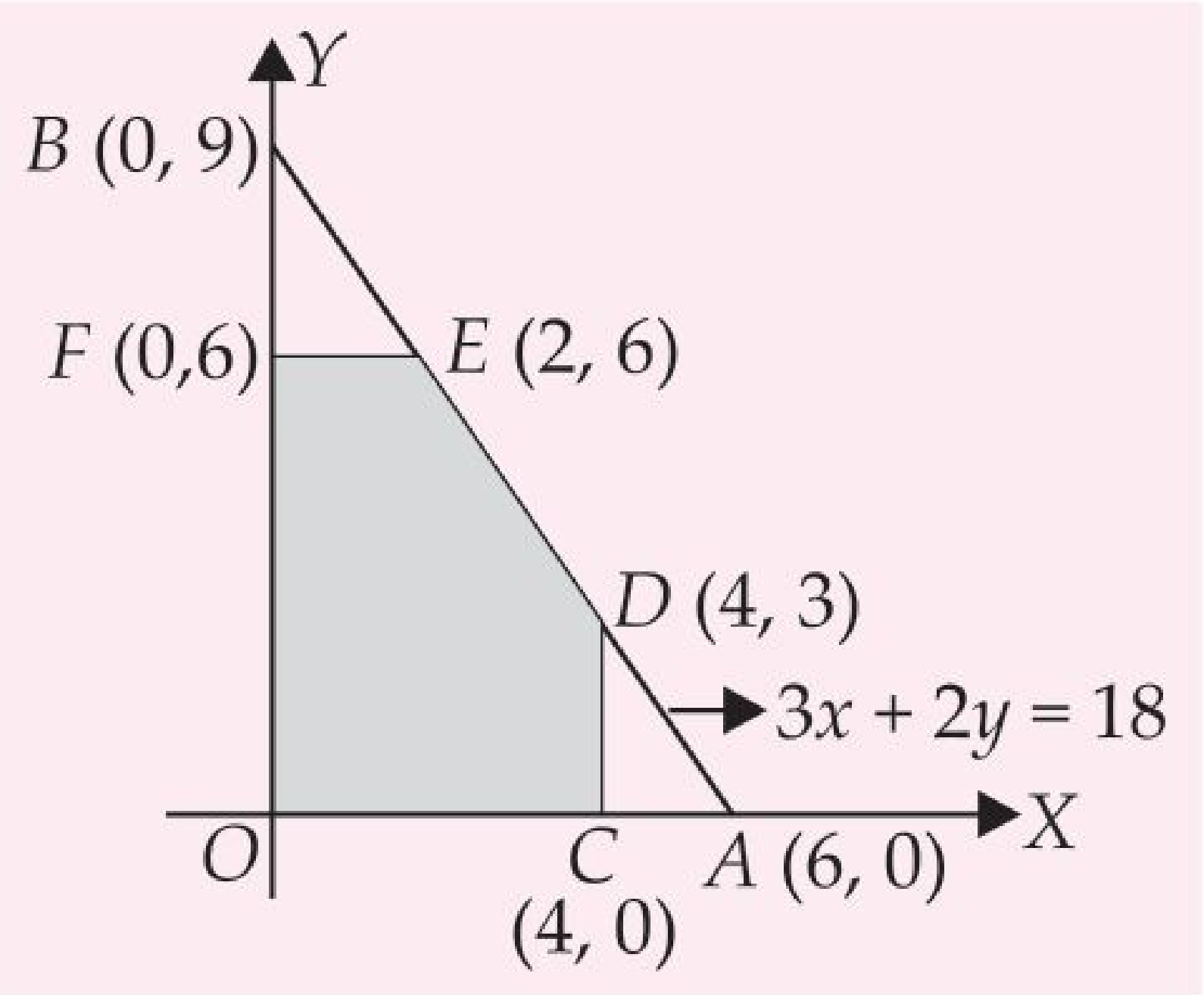
(a) [–1, 1] (b) [2, 3] (c) [3, 7] (d) [–7, –3]

22. If $x = 3\cos t - 2\cos^3 t$, $y = 3\sin t - 2\sin^3 t$, then $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$ is

(a) $16\sqrt{3}$ (b) $\sqrt{3}$

(c) $\frac{16}{3}$ (d) $\frac{-16}{2\sqrt{3}}$

23. In the given graph, the feasible region for a L.P.P. is shaded. The objective function $Z = 3x + 5y$ will be maximum at



(a) (2, 6) (b) (4, 3)
(c) (0, 6) (d) (4, 0)

24. If $f(x)=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, then the value of $(1-x^2)f'(x)-xf(x)$ is
(a) 0 (b) 1 (c) 2 (d) 3

25. If $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is a singular matrix, then the value of $5k-k^2$ is equal to
(a) 4 (b) 6 (c) –6 (d) –4

26. The function f defined by $f(x)=x^3-3x^2+5x+7$ is
(a) decreasing in R.
(b) increasing in R.
(c) decreasing in $(0,\infty)$ and increasing in $(-\infty,0)$.
(d) increasing in $(0,\infty)$ and decreasing in $(-\infty,0)$.

27. The principal value of $\cos^{-1}\left(\cos\dfrac{3\pi}{2}\right)$ is equal to
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{3\pi}{2}$ (c) $\dfrac{5\pi}{2}$ (d) $\dfrac{7\pi}{2}$

28. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $\det(A)=4$, then α is equal to
(a) 11 (b) 5 (c) 0 (d) 4

29. The function $f(x)=1-x^2$
(a) increases everywhere
(b) decreases in $(0,\infty)$
(c) increases in $(0,\infty)$
(d) None of these

30. Let $f:N\to N$ be defined by

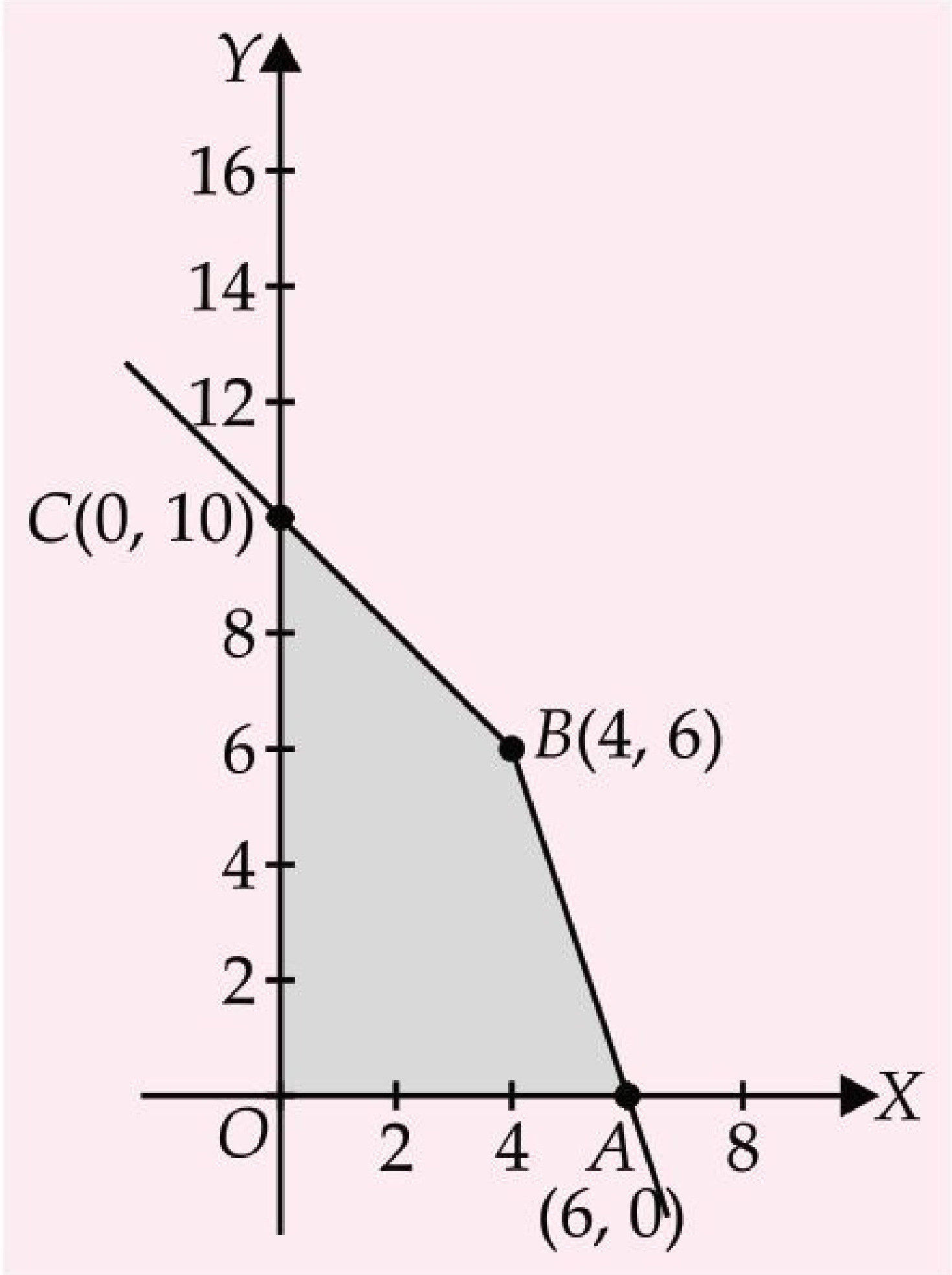
$$f(n)=\begin{cases}\dfrac{n+1}{2}, \text{ if } n \text{ is odd}\\ \dfrac{n}{2}, \text{ if } n \text{ is even}\end{cases} \quad \text{for all } n\in N.$$

The function f is
(a) one-one and onto
(b) one-one but not onto
(c) neither one-one nor onto
(d) onto but not one-one

31. The value of λ, for which the function $f(x)=\begin{cases}\lambda+(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$ is continuous at $x=0$, is
(a) 1 (b) –1
(c) 0 (d) None of these

32. If A is a 3×3 non-singular matrix such that $AA'=A'A$ and $B=A^{-1}A'$, then BB' equals
(a) I (b) B^{-1}
(c) $(B^{-1})'$ (d) $I+B$

33. In the given graph, the feasible region for a L.P.P. is shaded. The objective function $Z=4x+y$, will be maximum at



(a) (0, 0) (b) (6, 0)
(c) (4, 6) (d) (0, 10)

34. For the function $f(x)=xe^x$, the point
(a) $x=0$ is a maximum
(b) $x=0$ is a minimum
(c) $x=-1$ is a maximum
(d) $x=-1$ is a minimum

35. If $\begin{bmatrix}-2 & 5\\ 3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3\\ -1\end{bmatrix}$, then (x, y) is
(a) (1, 2) (b) (–1, 2)
(c) (1, –2) (d) (2, 1)

36. Find the value of $\tan^{-1}(1)+\cos^{-1}\left(\dfrac{-1}{2}\right)$.
(a) $\dfrac{\pi}{12}$ (b) $\dfrac{3\pi}{8}$ (c) $\dfrac{11\pi}{12}$ (d) $\dfrac{3\pi}{5}$

37. Let us define a relation R in $\mathbb{R}$ as aRb if $a \geq b$. Then, R is
(a) an equivalence relation
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) neither transitive nor reflexive but symmetric

38. If A_{ij} denotes the co-factor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then value of $a_{12}A_{32}$ is
(a) -90 (b) -66 (c) 102 (d) -55

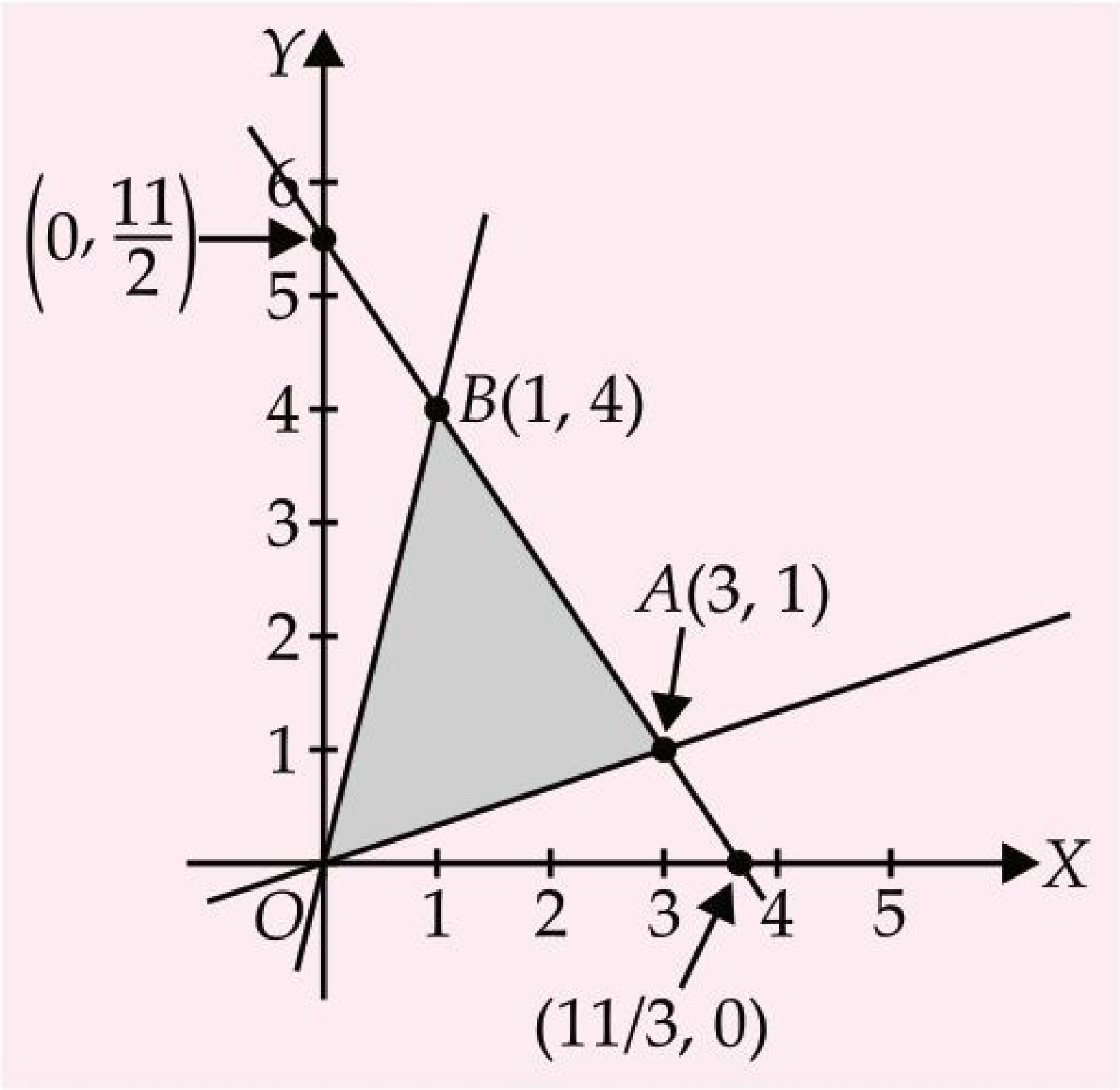
39. The normal at the point (1, 1) on the curve $2y + x^2 = 3$ is
(a) $x + y = 0$ (b) $x - y = 0$
(c) $x + y + 1 = 0$ (d) $-x - y = 0$

40. Let $Q = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$ and $x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$, then Q^3x is equal to
(a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

SECTION - C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

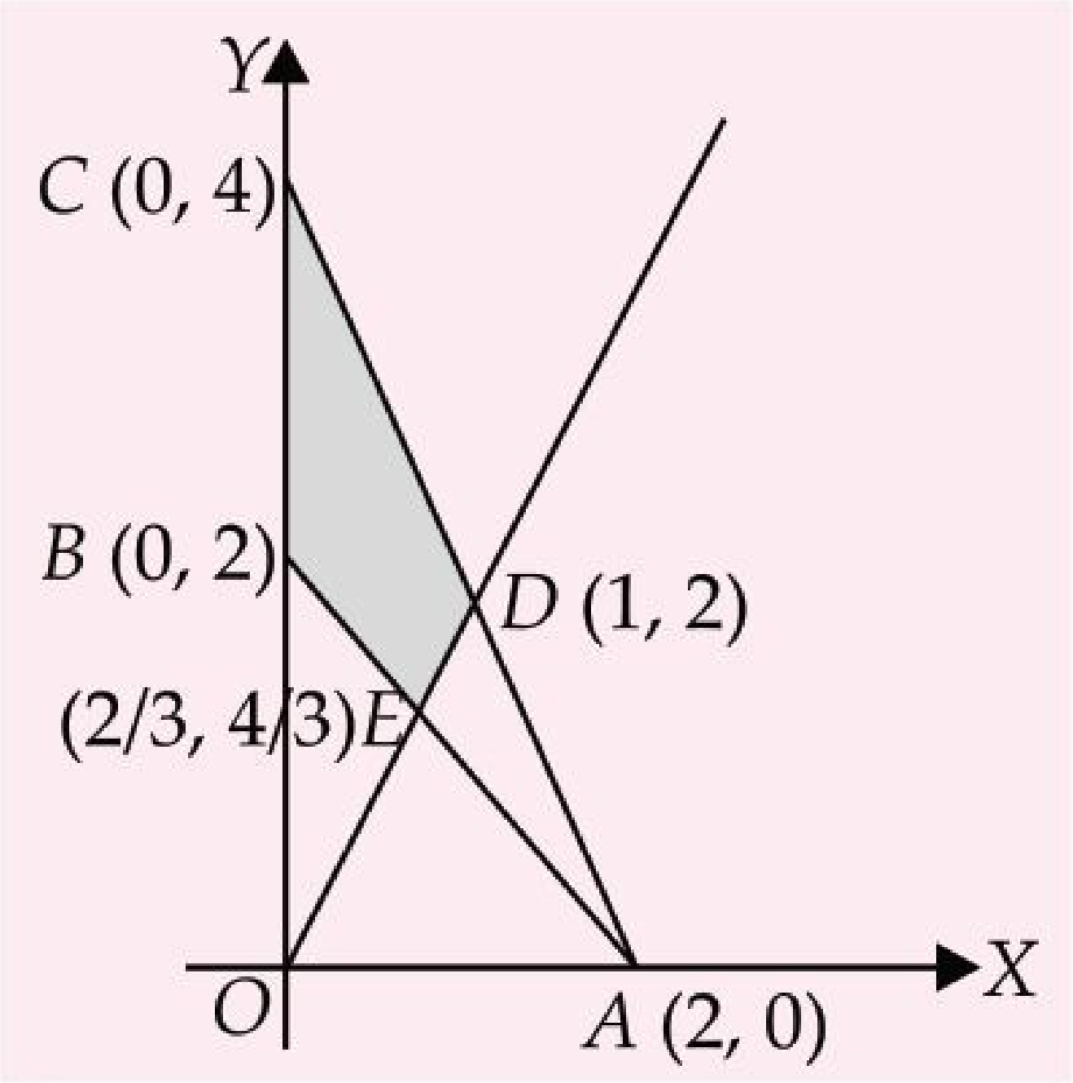
41. In the given graph, the feasible region for an L.P.P. is shaded. The maximum value of $z = 2x + 3y$ is



(a) 7 (b) 14 (c) 9 (d) 18

42. The slope of the normal to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) is
(a) $\frac{6}{7}$ (b) $-\frac{6}{7}$ (c) $\frac{7}{6}$ (d) $-\frac{7}{6}$

43. Based on the given shaded region as the feasible region in the graph, at which point is the objective function $z = 3x + 2y$ maximum?



(a) Point D (b) Point E
(c) Point B (d) Point C

44. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is
(a) $e^{-1/2}$ (b) $e^{1/e}$ (c) 1 (d) e^2

45. If $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x + iy$, then
(a) $x = 1, y = 1$ (b) $x = 0, y = 1$
(c) $x = 1, y = 0$ (d) $x = 0, y = 0$

CASE STUDY

Shobhit's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 200 ft of wire fencing.

Based on the above information, answer the following questions.

MtG

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46. To construct a garden using 200 ft of fencing, we need to maximise its

(a) volume (b) area

(c) perimeter (d) length of the side

47. If x denotes the length of side of garden perpendicular to brick wall and y denotes the length of side parallel to brick wall, then find the relation representing total amount of fencing wire.

(a) $x + 2y = 150$ (b) $x + 2y = 50$

(c) $y + 2x = 200$ (d) $y + 2x = 100$

48. Area of the garden as a function of x, say $A(x)$, can be represented as

(a) $200 + 2x^2$ (b) $x - 2x^2$

(c) $200x - 2x^2$ (d) $200 - x^2$

49. Maximum value of $A(x)$ occurs at x equals

(a) 50 ft (b) 30 ft (c) 26 ft (d) 31 ft

50. Maximum area of garden will be

(a) 2500 sq. ft (b) 4000 sq. ft

(c) 5000 sq. ft (d) 6000 sq. ft

SOLUTIONS

1. **(c) :** We have, $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)+\tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{6}\right)\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\sin^{-1}\left(\sin\frac{\pi}{6}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\sin^{-1}\left(\frac{1}{2}\right)+\frac{\pi}{6}=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$$

2. **(b) :** For continuity at $x = 0$, we have

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0) \qquad \text{...(i)}$$

Now, $\text{L.H.L.}=\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0} 6x^5=0$

Also, $\text{R.H.L.}=\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0}(2a+x)=2a$

From (i), we have $a = 0$

3. **(d) :** We have,

$$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 0 & 5 & 1\\ 0 & 2 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\\ x\end{bmatrix}=O$$

$$\Rightarrow \begin{bmatrix}1 & 5+5x & 2+x\end{bmatrix}\begin{bmatrix}1\\ 1\\ x\end{bmatrix}=O$$

$\Rightarrow$ $x^2 + 7x + 6 = 0$

$\Rightarrow$ $(x + 6)(x + 1) = 0 \Rightarrow x = -6$ or $x = -1$

4. **(c) :** $\begin{bmatrix}-1 & 3 & 0\\ 2 & 1 & 4\end{bmatrix}\times\begin{bmatrix}1 & 3\\ 0 & 2\\ -2 & -1\end{bmatrix}=\begin{bmatrix}-1 & 3\\ -6 & 4\end{bmatrix}$

5. **(c) :** We have, $f(x) = x^3 - 6x^2 - 36x + 7$

$\Rightarrow$ $f'(x) = 3x^2 - 12x - 36$

$= 3(x^2 - 4x - 12) = 3(x - 6)(x + 2)$

For strictly increasing function, $f'(x) > 0$

$\Rightarrow$ $x > 6$ or $x < -2$

$\therefore$ $f(x)$ is strictly increasing in $(-\infty, -2) \cup (6, \infty)$

6. **(b) :** $|A\cdot\text{adj}\,A|=\begin{vmatrix}10 & 0\\ 0 & 10\end{vmatrix}$

$\Rightarrow$ $|A|\,|\text{adj}\,A| = 100$ $\quad[\because\ |AB| = |A||B|]$

$\Rightarrow$ $|A||A|^{2-1} = 100$

$[\because\ |\text{adj}\,A| = |A|^{n-1}$, where n is the order of matrix $A]$

$\Rightarrow$ $|A|^2 = 100 \Rightarrow |A| = 10$

7. **(b) :** Here, $f(x) = x + 2$

Let $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 + 2 = x_2 + 2 \Rightarrow x_1 = x_2$

Let $y = x + 2$, $y \in Z$

Then, $x = y-2 \in Z\ \forall y \in Z$

Therefore, $f(x)$ is one-one and onto.

8. **(d) :** $B=\frac{1}{2}\left\{\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}-\begin{bmatrix}2 & 5\\ 3 & -1\end{bmatrix}\right\}$

$$=\frac{1}{2}\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

9. **(b) :** We have, $y = x^3 \Rightarrow \frac{dy}{dx}=3x^2$

Now, $3x^2 = y$ (By given condition)

$\Rightarrow$ $3x^2 = x^3 \Rightarrow x = 0, 3$

Thus, the two points are (0, 0) and (3, 27).

10. **(a) :** We know, domain of $\cos^{-1} x$ is $[-1, 1]$

$\therefore$ $-1 \le 2x - 1 \le 1 \Rightarrow 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$

Thus, the domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$.

11. **(b) :** R on the set {1, 2, 3} be defined by R = {(1, 2)}.

Clearly, R is transitive.

12. **(a) :** We have, $y=1-\frac{1}{x+1}+1+\frac{1}{x}$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{(1+x)^2}-\frac{1}{x^2}\Rightarrow\frac{d^2y}{dx^2}=\frac{-2}{(1+x)^3}+\frac{2}{x^3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\text{at } x=1}=-\frac{1}{4}+2=\frac{7}{4}$$

13. (a) : We have, $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$ and $C = (c_{ij})_{p \times q}$

Clearly, $BC = (b_{ij})_{n \times p} \times (c_{ij})_{p \times q} = (d_{ij})_{n \times q}$

$\therefore \quad (BC)A = (d_{ij})_{n \times q} \times (a_{ij})_{m \times n}$, which is possible only when $m = q$.

14. (a) : We have, $\tan^{-1}(x^2 + y^2) = \alpha$

$\Rightarrow \quad x^2 + y^2 = \tan\alpha$

Differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0 \;\Rightarrow\; \frac{dy}{dx} = \frac{-x}{y}$$

15. (d) : Clearly, we have $|A| = x - 2x(x - 1) = -9$

$\Rightarrow \quad x - 2x^2 + 2x = -9 \Rightarrow 2x^2 - 3x - 9 = 0$

$$\therefore \quad x = \frac{3 \pm \sqrt{9+72}}{4} = \frac{3 \pm 9}{4} \Rightarrow x = 3 \text{ or } \frac{-3}{2}$$

16. (d) : Slope of the line $x + 3y = 4$ is $\frac{-1}{3}$.

Since, normal to the curve $2x^2 - y^2 = 14$ at (x_1, y_1) is parallel to the line $x + 3y = 4$, therefore

slope of normal at $(x_1, y_1) = \frac{-1}{3}$

$$\Rightarrow \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} = \frac{-1}{3} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3$$

$$\Rightarrow \frac{2x_1}{y_1} = 3 \qquad \left[\because\; 4x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2x}{y}\right]$$

$$\Rightarrow \frac{x_1}{y_1} = \frac{3}{2}$$

From the given options, only option (d) satisfying this condition.

17. (c) : We have, $A = \begin{bmatrix} 0 & 3 \\ 4 & 5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 4a \\ 3b & 60 \end{bmatrix}$

From these two matrices, we get

$ka_{22} = 60 \Rightarrow k(5) = 60 \qquad [\because a_{22} = 5]$

$\Rightarrow \quad k = 12$

Now, by putting $k = 12$, we get $a = 9$ and $b = 16$.

18. (b) : We have, $y = \log(\log x) \Rightarrow \frac{dy}{dx} = \frac{1}{x \log x}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-1}{(x\log x)^2}\left(x \cdot \frac{1}{x} + \log x \cdot 1\right) = \frac{-(1 + \log x)}{(x \log x)^2}$$

19. (c) : The corner points of the feasible region are $A(4, 3)$, $B(0, 6)$ and $C(0, 5)$.

The value of $z = 2x + 5y$ at point A, B and C is $z(A) = 23$, $z(B) = 30$ and $z(C) = 25$

$\therefore \quad$ Minimum value of $z = 23$ at A.

20. (b) : We have, $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$

$\Rightarrow \quad f'(x) = 3x^2 + 12x + (9 + 2k)$

$= 3(x^2 + 4x) + 9 + 2k = 3(x^2 + 4x + 4) - 12 + 9 + 2k$

$$= 3(x + 2)^2 - 3 + 2k > 0 \quad \left[\because\; k > \frac{3}{2}\right]$$

Thus, $f(x)$ is strictly increasing $\forall\, x \in R$.

21. (d) : $f(x) = \sin^{-1}\left(\frac{x+5}{2}\right) \;\Rightarrow\; -1 \le \frac{x+5}{2} \le 1$

$\Rightarrow \quad -2 - 5 \le x \le 2 - 5 \Rightarrow -7 \le x \le -3$

$\therefore \quad D(f(x)) = [-7, -3]$

22. (b) : $\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t$,

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t \cos t$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{t = \frac{\pi}{6}} = \frac{3\cos\pi/6 - 6\sin^2\pi/6 \cdot \cos\pi/6}{-3\sin\pi/6 + 6\cos^2\pi/6 \cdot \sin\pi/6}$$

$$= \frac{3 \cdot \frac{\sqrt{3}}{2} - 6 \cdot \frac{1}{4} \cdot \frac{\sqrt{3}}{2}}{-3 \cdot \frac{1}{2} + 6 \cdot \frac{3}{4} \cdot \frac{1}{2}} = \frac{3\sqrt{3}/4}{3/4} = \sqrt{3}$$

23. (a) : We have, $Z = 3x + 5y$

$\therefore \quad Z(O) = 0$

$Z(C) = 3 \times 4 + 5 \times 0 = 12$

$Z(D) = 3 \times 4 + 5 \times 3 = 27$

$Z(E) = 3 \times 2 + 5 \times 6 = 36$

$Z(F) = 3 \times 0 + 5 \times 6 = 30$

$\therefore \quad$ Maximum value of Z is 36 at point $E(2, 6)$.

24. (b) : We have, $f(x) = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - \sin^{-1} x \times \frac{1}{2\sqrt{1-x^2}} \times (-2x)}{1 - x^2}$$

$\Rightarrow \quad (1 - x^2) f'(x) - x f(x) = 1$

25. (a) : Since matrix A is singular, so $|A| = 0$

$$\Rightarrow \begin{vmatrix} 2-k & 2 \\ 1 & 3-k \end{vmatrix} = 0 \Rightarrow (2 - k)(3 - k) - 2 = 0$$

$\Rightarrow \quad k^2 - 5k + 4 = 0 \Rightarrow 5k - k^2 = 4$

26. **(b) :** We have, $f(x) = x^3 - 3x^2 + 5x + 7$

$\Rightarrow \; f'(x) = 3x^2 - 6x + 5$

$\Rightarrow \; f(x) = 3(x-1)^2 + 2 > 0 \quad \forall\, x \in R$

So, $f(x)$ is increasing in R.

27. **(a) :** We know, cosine of odd multiple of $\frac{\pi}{2}$ is 0

$\therefore \quad \cos\frac{3\pi}{2} = 0$

$\therefore \quad \cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}(0) = \frac{\pi}{2}$

$$\left[\because\ \cos\frac{\pi}{2} = 0 \text{ and } \frac{\pi}{2} \in [0, \pi]\right]$$

28. **(a) :** We have, $\det(P) = \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$

$\Rightarrow \quad \det(P) = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6)$

$\Rightarrow \quad \det(\operatorname{adj} A) = 2\alpha - 6$

Also, $\det(\operatorname{adj} A) = (\det A)^2$

$\Rightarrow \quad 2\alpha - 6 = 16$

$\Rightarrow 2\alpha = 22$

$\therefore \quad \alpha = 11$

29. **(b) :** We have, $f(x) = 1 - x^2$

$\Rightarrow f'(x) = -2x$

Since $f'(x) < 0$ for $x \in (0, \infty)$

$\therefore \quad f(x)$ decreases in $(0, \infty)$

30. **(d) :** Here $f(1) = \frac{1+1}{2} = 1, \; f(2) = \frac{2}{2} = 1$

$\Rightarrow \quad f(1) = f(2)$ but $1 \neq 2$

$\Rightarrow f$ is not one-one.

But f is onto because range of $f = N$

$[\because$ For any $x \in N, 2x \in N$ such that $f(2x) = \frac{2x}{2} = x]$

$\Rightarrow \quad f$ is onto.

31. **(a) :** We have, $f(x) = \begin{cases} \lambda + (x^2 - 2x), & \text{if } x \le 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} [\lambda + (x^2 - 2x)] = \lambda + 0 = \lambda$$

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} (4x + 1) = 1$$

Since $f(x)$ is continuous at $x = 0$.

$\therefore \quad \lambda = 1.$

32. **(a) :** $BB' = (A^{-1}A')(A^{-1}A')'$

$= (A^{-1}A')((A')'(A^{-1})') = (A^{-1}A')(A(A^{-1})')$

$= A^{-1}(A'A)(A')^{-1} = A^{-1}(AA')(A')^{-1}$

$= (A^{-1}A)(A')(A')^{-1} = I \cdot I = I$

33. **(b) :** We have, $Z = 4x + y$

Corner Points	**Value of Z**
(0, 0)	0
(6, 0)	24 (Maximum)
(4, 6)	22
(0, 10)	10

$\therefore \quad$ Maximum of $Z = 24$ at $(6, 0)$.

34. **(d) :** We have, $f(x) = xe^x$

$\Rightarrow \quad f'(x) = xe^x + e^x$

To be maximum or minimum, $f'(x) = 0$

$\Rightarrow \quad (x + 1)e^x = 0$

$\Rightarrow \quad x = -1 \qquad (\because\ e^x \neq 0)$

$f''(x) = xe^x + e^x + e^x$

$\therefore \quad f''(-1) = -e^{-1} + e^{-1} + e^{-1} = e^{-1} > 0$

So, $x = -1$ is a minimum

35. **(d) :** We have, $\begin{bmatrix} -2 & 5 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} -2x + 5y \\ 3x - y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$\Rightarrow \quad -2x + 5y = 1$ ...(i), $3x - y = 5$ ...(ii)

From (i) and (ii), we have

$-2x + 5(3x - 5) = 1$

$\Rightarrow 13x = 26 \Rightarrow x = 2$

Substituting $x = 2$ in (ii), we get $y = 6 - 5 = 1$

Hence, $(x, y) = (2, 1)$

36. **(c) :** Let $\tan^{-1}(1) = x \Rightarrow 1 = \tan x$

$\Rightarrow \quad 1 = \tan\left(\frac{\pi}{4}\right)$, where $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y \Rightarrow -\frac{1}{2} = \cos y$

$\Rightarrow \quad -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3},$

where $\frac{2\pi}{3} \in [0, \pi]$

So, $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$

37. **(b) :** Since, $a \ge a \Rightarrow aRa$

$\Rightarrow R$ is reflexive.

Let $aRb \Rightarrow a \ge b \Rightarrow b \le a$

$\therefore \quad R$ is not symmetric.

If aRb and bRc

Then $a \ge b$ and $b \ge c$

$\Rightarrow \quad a \ge c \Rightarrow aRc$

Hence, R is transitive.

38. (b) : $M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22$

$\Rightarrow \quad A_{32} = -M_{32} = 22 \Rightarrow a_{12}A_{32} = (-3)(22) = -66$

39. (b) : We have, $2y + x^2 = 3$ ...(i)
Differentiating (i) w.r.t. x, we get

$$2\frac{dy}{dx} + 2x = 0 \Rightarrow \frac{dy}{dx} = -x$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(1,1)} = -1$$

$\therefore$ Slope of normal = 1
$\therefore$ The equation of normal is
$y - 1 = 1 \cdot (x - 1) \Rightarrow x - y = 0$

40. (c) : We have, $Q = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$Q^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ and } Q^3 = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\therefore \quad Q^3 x = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

41. (b) : The corner points of the feasible region are $O(0, 0)$, $A(3, 1)$, $B(1, 4)$.
$z = 2x + 3y$ at O, A and B is
The value of $z(O) = 0$, $z(A) = 9$ and $z(B) = 14$
$\therefore$ Maximum value of $z = 14$ at B.

42. (d) : $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$

$$\therefore \quad \frac{dx}{dt} = 2t + 3 \text{ and } \frac{dy}{dt} = 4t - 2$$

$$\text{Now, } \frac{dy}{dx} = \frac{4t - 2}{2t + 3}$$

At the point $(2, -1)$, $t = 2$

$$\therefore \quad \left[\frac{dy}{dx}\right]_{t=2} = \frac{8 - 2}{4 + 3} = \frac{6}{7}$$

$$\therefore \quad \text{Slope of normal} = -\frac{7}{6}$$

43. (d) : The feasible region is quadrilateral $BCDE$.
Now, $z(B) = 2 \times 2 = 4$
$z(C) = 2 \times 4 = 8$ (Maximum)
$z(D) = 3 \times 1 + 2 \times 2 = 7$
$z(E) = 3 \times \frac{2}{3} + 2 \times \frac{4}{3} = \frac{14}{3}$
$\therefore$ Maximum value of $z = 8$ at point C.

44. (b) : Let $y = \left(\frac{1}{x}\right)^{2x^2} \Rightarrow \log y = 2x^2 \log\frac{1}{x}$

Differentiating w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = 4x \log\frac{1}{x} + 2x^2\left(-\frac{1}{x}\right)$$

For maxima/minima, put $\frac{dy}{dx} = 0$

$\Rightarrow \quad -4x \log x - 2x = 0 \Rightarrow 2 \log x + 1 = 0 \Rightarrow x = e^{-1/2}$

$$\text{Now, } \left[\frac{d^2y}{dx^2}\right]_{x = e^{-1/2}} < 0$$

$\therefore \quad y_{\max} = e^{1/e}$

45. (d) : We have, $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = (x + iy)$

$\Rightarrow \quad 3i(9i^2 + 9) + 9i(2i + 10) + (18 - 90i) = x + iy$
$\Rightarrow \quad 3i(-9 + 9) + 18i^2 + 90i + 18 - 90i = x + iy$
$\Rightarrow \quad -18 + 18 = x + iy \Rightarrow 0 + 0i = x + iy$
On comparing, we get $x = 0$ and $y = 0$

46. (b) : To create a garden using 200 ft fencing, we need to maximise its area.

47. (c) : Required relation is given by $2x + y = 200$.

48. (c) : Area of garden as a function of x can be represented as
$A(x) = x \cdot y = x(200 - 2x) = 200x - 2x^2$

49. (a) : $A(x) = 200x - 2x^2 \Rightarrow A'(x) = 200 - 4x$
For the area to be maximum, $A'(x) = 0$
$\Rightarrow \quad 200 - 4x = 0 \Rightarrow x = 50$ ft.
Also, $A''(x) = -4 < 0$
Thus, $x = 50$ is point of maximum.

50. (c) : Maximum area of the garden
$= 200(50) - 2(50)^2 = 10000 - 5000 = 5000$ sq. ft

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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Application of Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

1. The tangent at any point P on the curve $\sqrt{x}+\sqrt{y}=2$ meets the coordinate axes at A and B. The sum of the intercepts OA and OB is

(a) 1 (b) 2 (c) 3 (d) 4

2. The function $f(x)=\frac{1}{8}\ln x-2x+x^2$, $x>0$ attains maximum at $x=$

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$ (c) $\frac{2-\sqrt{3}}{4}$ (d) $\frac{2+\sqrt{3}}{4}$

3. Let $f(x)=ax^2-8x+b$ and $3^{\sqrt{\log_3 7}}$ and $7^{\sqrt{\log_7 3}}$ are roots of the equation $f(x)=0$, then maximum value of $f(1)$ is, (If a, b are positive integers)

(a) 8 (b) 9 (c) 10 (d) 11

4. Let $y=f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}\,dt$, then

(a) $f(x)$ has local maximum at $x=-1$ and $x=1$

(b) $f(x)$ increases in $(-2,-1)\cup(0\ 1)$

(c) $f(x)$ has local minima at $x=-2, 0, 2$

(d) All of these including $f(x)$ decreases in $(-1,0)\cup(1,2)$

5. The minimum value of $x^2-x+1+\sin x$ is given by

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{1}{4}$ (d) $\frac{7}{4}$

6. If $A=x^2+1/x^2$, $B=x-1/x$, then minimum value of $\frac{A}{B}$ equals

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) None of these

One or More than One Option(s) Correct Type

7. If the tangent to the curve $2y^3=ax^2+x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes, (where $\alpha^2+\beta^2=61$), then the value of a is

(a) -30 (b) 10

(c) 20 (d) 30

8. Let $f(x)=p(x)+q(x)$, where $p(x)=\frac{\sin x}{x}$, $q(x)=\frac{x}{\tan x}\ \forall\ x\in\left(0,\frac{\pi}{2}\right)$ then

(a) $p(x)$ increases in $\left(0,\frac{\pi}{2}\right)$ and $q(x)$ decreases in $\left(0,\frac{\pi}{2}\right)$

(b) $p(x)$ decreases in $\left(0,\frac{\pi}{2}\right)$ and $q(x)$ increases in $\left(0,\frac{\pi}{2}\right)$

(c) both $p(x)$ & $q(x)$ decreasing in $\left(0,\frac{\pi}{2}\right)$

(d) the minimum value of $f(x)$ when $x\to\frac{\pi}{2}$ is $\frac{2}{\pi}$ & the maximum value of $f(x)$ is 2 when $x\to 0$.

9. Which of the following pair(s) of family is/are orthogonal?

(a) $16x^2+y^2=c$ and $y^{16}=kx$

(b) $y=x+ce^{-x}$ and $x+2=y+ke^{-y}$

(c) $y=cx^2$ and $x^2+2y^2=k$

(d) $x^2-y^2=c$ and $xy=k$

where c and k are arbitrary constants.

10. The curve $y=ax^3+bx^2+cx$ is inclined at 45° to x-axis at (0,0) but it touches x-axis at (1,0), then

(a) $f'(1)=0$ (b) $f''(1)=2$

(c) $f'''(2)=12$ (d) $f(2)=2$

11. The abscissa of the point on the curve $\sqrt{xy} = a + x$, the tangent at which cuts off equal intercepts from the coordinate axes is

(a) $-\frac{a}{\sqrt{2}}$ (b) $\frac{a}{\sqrt{2}}$

(c) $-a\sqrt{2}$ (d) $a\sqrt{2}$

12. The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at the point $P(0, -3)$ is tangent to the curve at the point(s)

(a) $(2, -44)$ (b) $(-2, 38)$

(c) $(1, -1)$ (d) $(-1, -5)$

Comprehension Type

Paragraph for Q. No. 13 to 15

Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x \to 1} \frac{f(x)}{(x-1)^2} = 1$, $f'(0) = -6$, $f'(2) = 6$.

13. The minimum value of $f(x)$ is

(a) 0 (b) 1 (c) -1 (d) -2

14. The length of subtangent of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

15. $f''(0) =$

(a) 6 (b) 8 (c) 12 (d) 14

Matrix Match Type

16. Match the following :

Column-I		Column-II	
P.	The maximum value of $f(x) = 9x(x-1)^2$, $0 \le x \le 2$ is	1.	-1
Q.	The minimum value of $f(x) = \frac{x^2-1}{x^2+1}$ is	2.	$\frac{4}{3}$
R.	The maximum value of $f(x) = \frac{x^2+1}{x^2-1}$, $x \in R - \{-1, 1\}$ is	3.	-2
S.	The maximum value of $f(x) = \frac{x^2+1}{x}$, $x \in R - \{0\}$ is	4.	2

	P	Q	R	S
(a)	2	1	1	3
(b)	3	4	2	1
(c)	1	3	4	2
(d)	4	1	3	2

Numerical Answer Type

17. The area of the largest circle with centre $\left(\frac{1}{2}, 0\right)$ inscribed in the ellipse $x^2 + 2y^2 = 2$ is $\frac{m\pi}{n}$, where m and n are relatively prime positive integers. Then $m + n$ is equal to ________.

18. An aeroplane is flying horizontally at a height of $\frac{2}{3}$ km with a velocity of 15 km/hr. The rate (in km/hr) at which it is receding from a fixed point on the ground which it passed over 2 minutes ago is ________.

19. Let $0 < a < b < \frac{\pi}{2}$. If $f(x) = \begin{vmatrix} \tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \end{vmatrix}$, then minimum possible number of roots of $f'(x) = 0$ in (a, b) is ________.

20. For a twice differentiable function $f(x)$, $g(x)$ is defined as $g(x) = (f'(x))^2 + f(x)f''(x)$, on $[a, e]$. If $a < b < c < d < e$, $f(a) = 0$, $f(b) = 2$, $f(c) = -1$, $f(d) = 2$, $f(e) = 0$, then the minimum number of roots of the equation $g(x) = 0$ is ________.

◆◆

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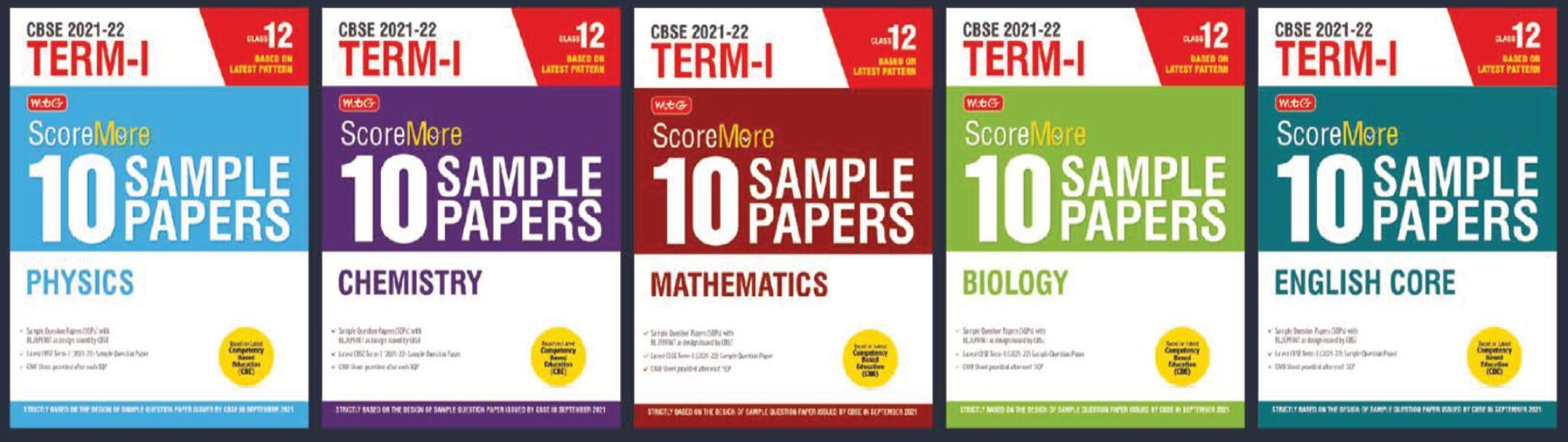
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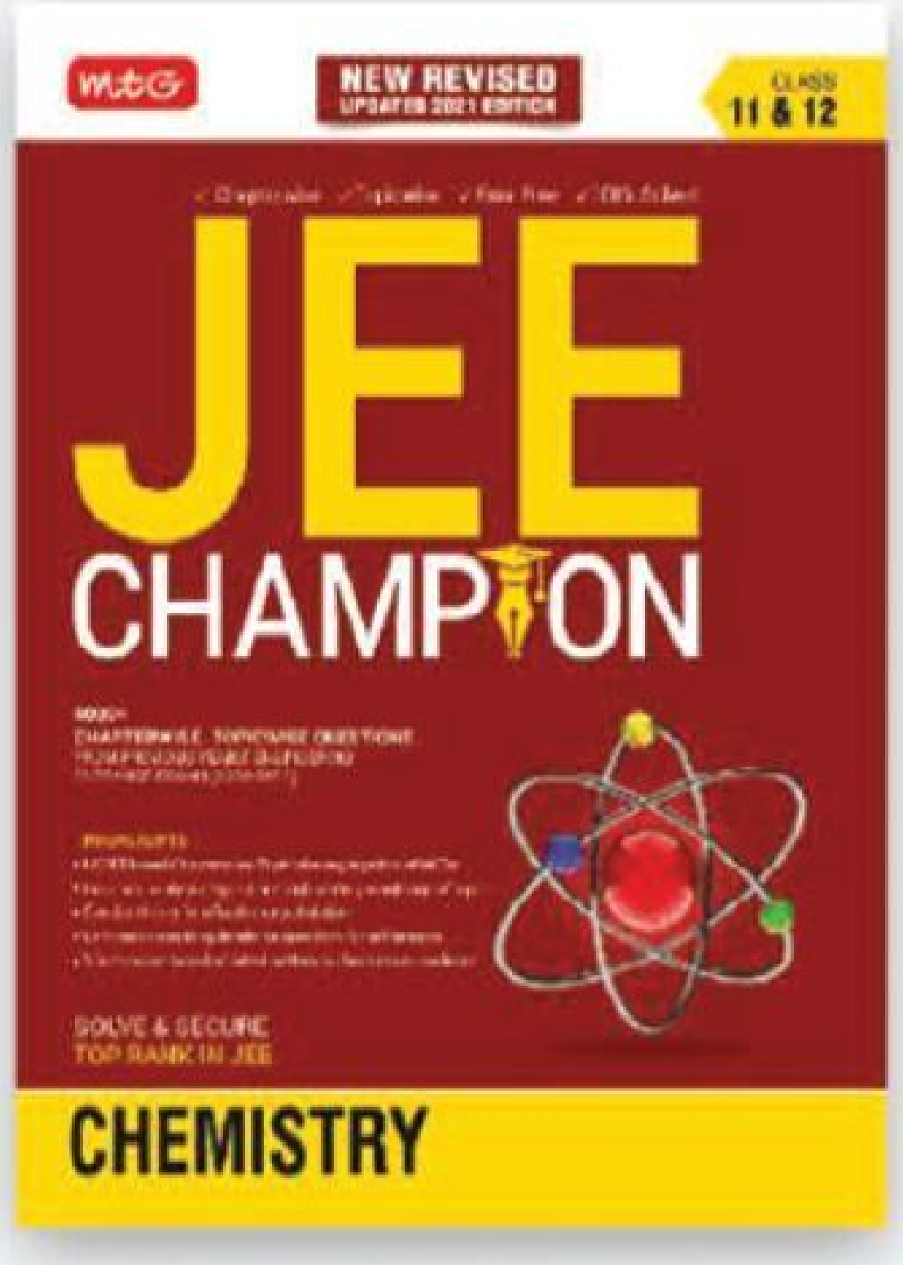
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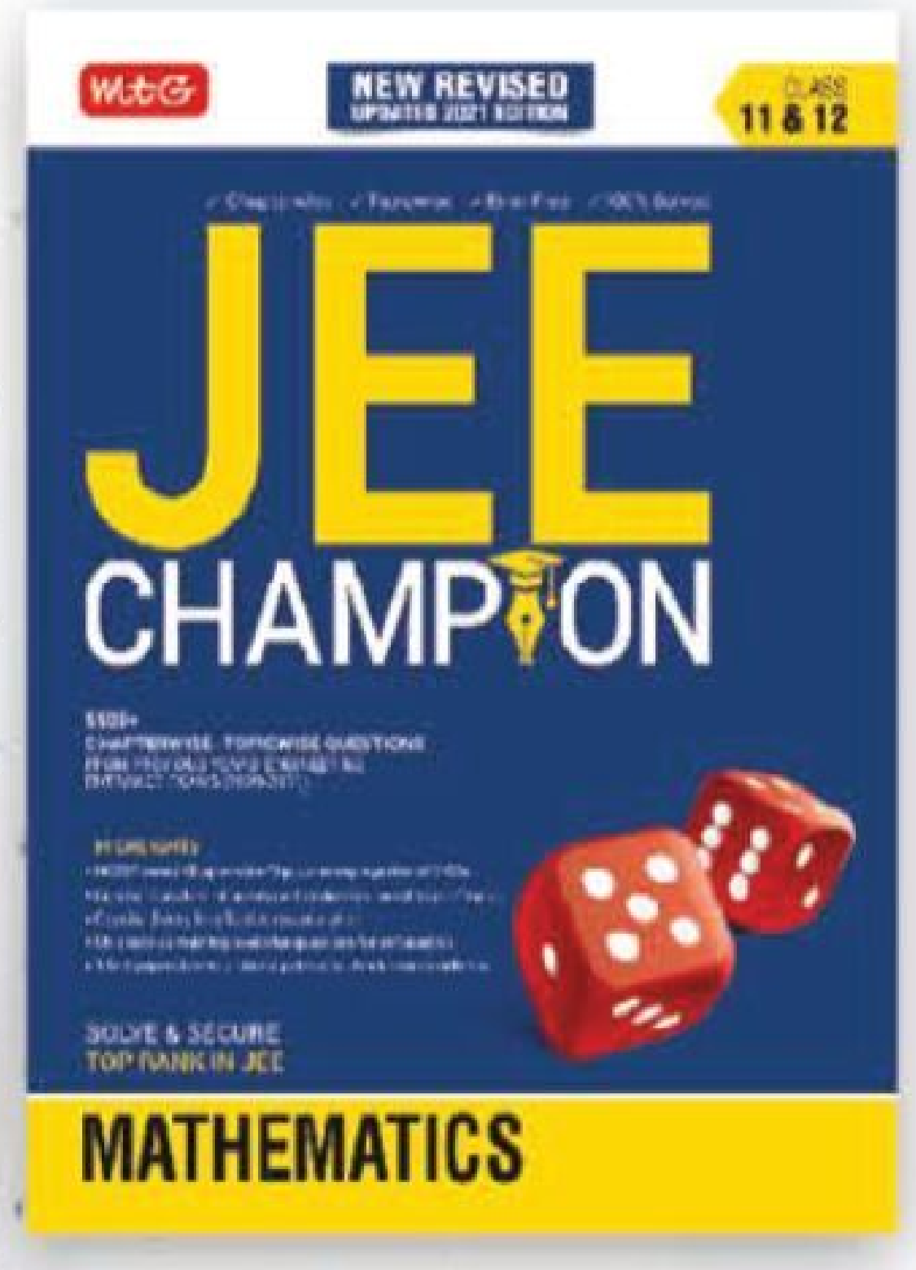
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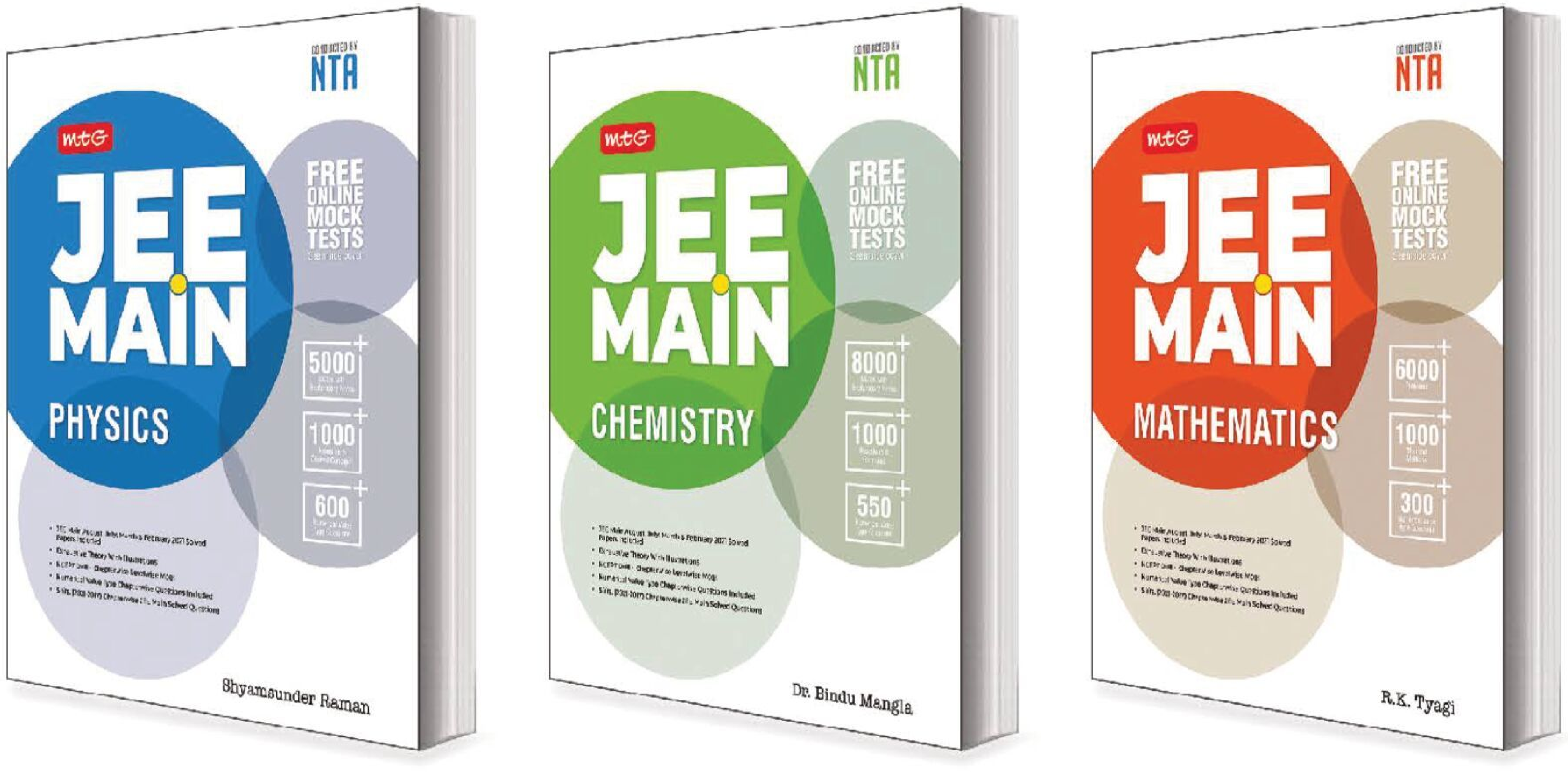

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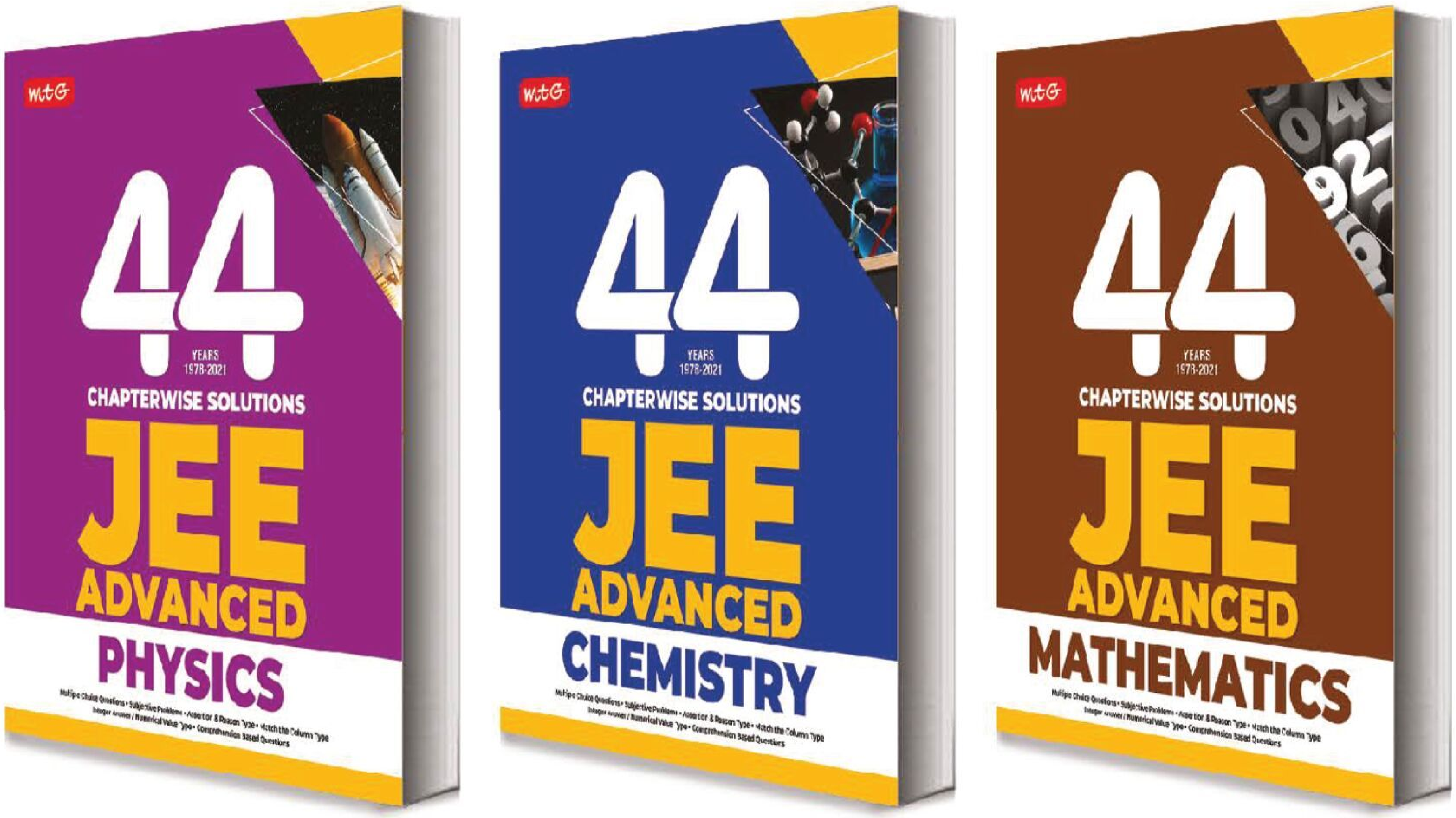
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